Declining CO_2 price paths

Kent Daniel
kd2371@columbia.edu
kentdaniel.net

Kepos Capital

Bob Litterman
bob@keposcapital.com

NYU

Gernot Wagner
gwagner@nyu.edu
gwagner.com

NBER WORKING PAPER SERIES

APPLYING ASSET PRICING THEORY TO CALIBRATE THE PRICE OF CLIMAT RISK

Kent D. Daniel
Robert B. Litterman
Gernot Wagner

Working Paper 22795
http://www.nber.org/papers/w22795

PNAS

Declining CO_2 price paths

Kent D. Daniel[a,b,1], Robert B. Litterman[c,1], and Gernot Wagner[d,1,2,3]

[a]Columbia Business School, New York, NY 10027; [b]National Bureau of Economic Research, Cambridge, MA 02138; [c]Kepos Capital, New York, NY 10018; and [d]Harvard University Center for the Environment, Cambridge, MA 02138

Edited by Jose A. Scheinkman, Columbia University, New York, NY, and approved September 9, 2019 (received for review October 9, 2018)

Pricing greenhouse-gas (GHG) emissions involves making trade-offs between consumption today and unknown damages in the (distant) future. While decision making under risk and uncertainty is the forte of financial economics, important insights from pricing financial assets do not typically inform standard climate–economy models. Here, we introduce EZ-Climate, a simple recursive dynamic asset pricing model that allows for a calibration of the carbon dioxide (CO_2) price path based on probabilistic assumptions around climate damages. Atmospheric CO_2 is the "asset" with a negative expected return. The economic model focuses on society's willingness to substitute consumption across time and across uncertain states of nature, enabled by an Epstein–Zin (EZ) specification that delinks preferences over risk from intertemporal substitution. In contrast to most modeled CO_2 price paths, EZ-Climate suggests a high price today that is expected to decline over time as the "insurance" value of mitigation declines and technological change makes emissions cuts cheaper. Second, higher risk aversion increases both the CO_2 price and the risk premium relative to expected damages. Lastly, our model suggests large costs associated with delays in pricing CO_2 emissions. In our base case, delaying implementation by 1 y leads to annual consumption losses of over 2%, a cost that roughly increases with the square of time per additional year of delay. The model also makes clear how sensitive results are to key inputs.

climate risk | asset pricing | cost of carbon

For over 25 y, the dynamic integrated climate–economy (DICE) model (1–3) has been the standard tool for analyzing CO_2 emissions-reductions pathways, and for good reason. One attraction is its simplicity, turning a "market failure on the greatest scale the world has seen" (4) and "the mother of all externalities" (5) into a model involving fewer than 20 main equations, 3 representing the climate system (6). DICE has spawned many variants (7). It has also helped set the tone for what many consider "optimal" CO_2 price paths. The core trade-off between economic consumption and climate damages leads to relatively low CO_2 prices today rising over time.

DICE and models like it have well-known limitations, including how they represent climate risk and uncertainty (7–15). DICE, for example, is not an optimal-control model, as commonly understood by economists employing modern dynamic economic analysis, even though it lends itself to those extensions (9–12). The underlying structure all but prescribes a rising CO_2 price path over time.

One important limitation is the form of the utility function. Constant relative risk aversion (CRRA) preferences, standard in most climate–economy models (1, 7, 16), assume that economic agents have an equal aversion to variation in consumption across states of nature and over time. Evidence from financial markets suggests that this is not the case (17). The risk premium (RP) of equities over bonds points to a fundamental difference in how much society is willing to pay to substitute consumption risk across states of nature compared to over time (18, 19). Some have explained the discrepancy by allowing for extreme events (20–22), and others have looked to more flexible preference specification fo... (23–26) or both (27). Our own preference specification ... Epstein and Zin (EZ) (24, 25).

EZ Preferences

Here, we use EZ preferences and focus on climate uncertainties. We approach climate change as an asset pricing problem with atmospheric CO_2 as the "asset." The value of an investment in reducing CO_2 emissions depends on the state of nature, represented by its fragility θ_t. That, in turn, helps determine the discount rate applied to the damages that would have occurred without the investment.

Our representative agent maximizes a recursive utility U_t based on consumption c_t and expectations E_t over future utility for times $t \in \{0, 1, 2, \ldots, T-1\}$:

$$U_t = \left[(1-\beta)c_t^{\rho} + \beta\left(E_t\left[U_{t+1}^{\alpha}\right]\right)^{\frac{\rho}{\alpha}}\right]^{\frac{1}{\rho}}. \quad [1]$$

Parameters α and ρ measure the agent's willingness to substitute consumption across states of nature and across time, respectively. (See *Methods* for the final-period utility U_T and further derivations.) CRRA preferences are a special case, with $\alpha = \rho$. Unlike with CRRA, Eq. **1** implies that CO_2 prices no longer collapse to zero with increasing risk aversion (RA) and equity risk premia (Fig. 1*A*). The same goes for the portion of CO_2 prices explained by RA (Fig. 1*B*).

EZ preferences have since found their way into the climate–economic literature (9–12, 28–35). Some have embedded EZ into DICE (28, 35), and others employ supercomputers to solve (9–12). The complexity typically does not allow for analytic solutions (34). We here follow a simple binomial-tree model with a long history in financial modeling (36). It is precisely this modeling choice—standard in financial economics but novel to climate–economic applications—that leads to our fundamentally differing CO_2 price paths. Mitigating climate risk provides

Significance

Risk and uncertainty are important in pricing climate damages. Despite a burgeoning literature, attempts to marry insights from asset pricing with climate economics have largely failed to supplement—let alone supplant—decades-old climate–economy models, largely due to their analytic and computational complexity. Here, we introduce a simple, modular framework that identifies core trade-offs, highlights the sensitivity of results to key inputs, and helps pinpoint areas for further work.

Author contributions: K.D.D., R.B.L., and G.W. designed research, performed research, contributed new analytic tools, and wrote the paper.

The authors declare no competing interest.

This article is a PNAS Direct Submission.

[1]K.D.D., R.B.L., and G.W. contrib...

[2]Present address: ... York, NY 1003...

ECONOMIC SCIENCES

gwagner.com/ezclimate

Four novel conclusions:

1. Increased risk aversion *increases* the CO_2 price
   in contrast to most standard models employing power utility functions, where increased risk aversion implies a higher discount rate implies a lower CO_2 price

2. CO_2 price *declines* over time
   in contrast to most standard models with the exception of Ulph & Ulph (1994) [producer behavior], Acemoglu et al (2012) [shift from “dirty” to “clean”], Lemoine & Rudik (2017) [inertia]

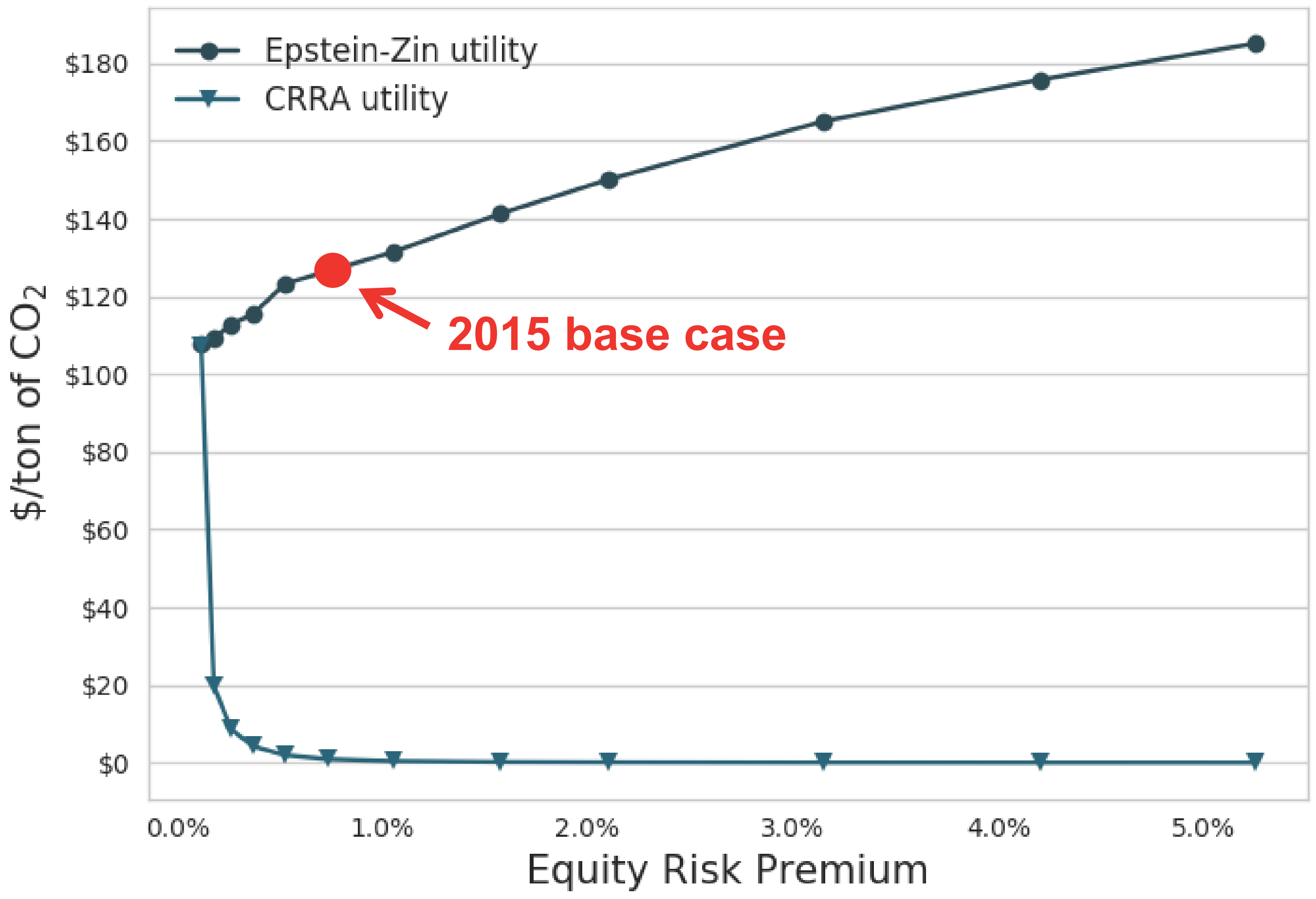
3. Increased risk aversion increases risk premium relative to expected damages
   in contrast to standard models due to their use of power utility functions and (typically) lack of possibility for ‘catastrophic’ damages

4. Enormous social costs of delay
   in contrast to most standard models, which often estimate cost of delay based on (rising) ‘optimal’ CO_2 price over time in any given year (e.g. Nordhaus 2017, Changes in the DICE model, 1992 – 2017)

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

1 Standard utility specifications misrepresent (climate) risk

Constant Relative Risk Aversion (CRRA) utility conflates risk across time and across states of nature



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Four novel conclusions:

1. Increased risk aversion *increases* the CO_2 price

   in contrast to most standard models employing power utility functions, where increased risk aversion implies a higher discount rate implies a lower CO_2 price

2. CO_2 price *declines* over time

   in contrast to most standard models with the exception of Ulph & Ulph (1994) [producer behavior], Acemoglu et al (2012) [shift from “dirty” to “clean”], Lemoine & Rudik (2017) [inertia]

3. Increased risk aversion increases risk premium relative to expected damages

   in contrast to standard models due to their use of power utility functions and (typically) lack of possibility for ‘catastrophic’ damages

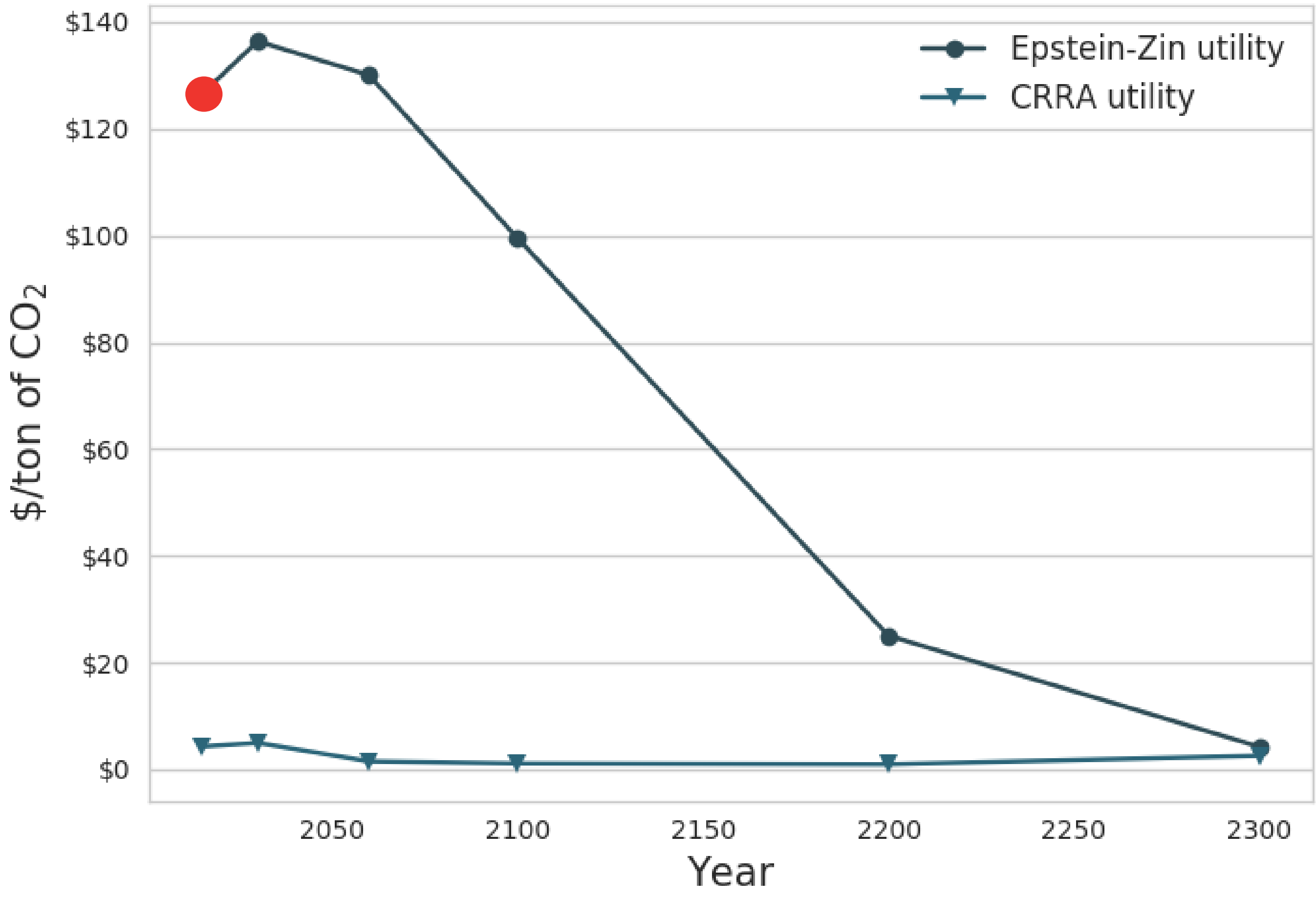
4. Enormous social costs of delay

   in contrast to most standard models, which often estimate cost of delay based on (rising) ‘optimal’ CO_2 price over time in any given year (e.g. Nordhaus 2017, Changes in the DICE model, 1992 – 2017)

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

2 CO_2 price declines over time

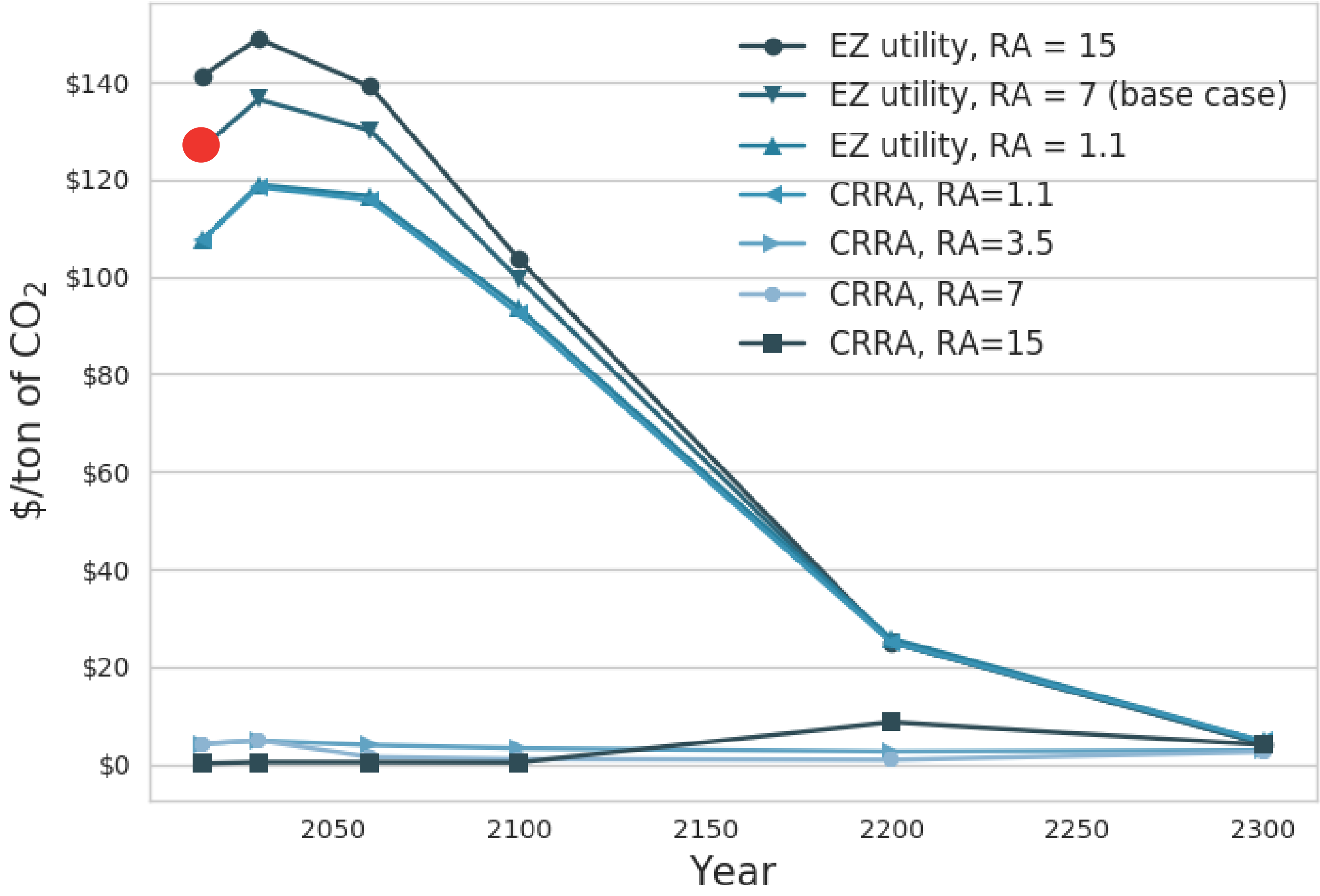
Starts $>100, declines as uncertainties clear up



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

2 CO_2 price sensitive to utility specification for ‘reasonable’ RA values

No difference between CRRA and EZ utility at RA=1.1, large differences for RA>~3



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Four novel conclusions:

1. Increased risk aversion *increases* the CO_2 price

   in contrast to most standard models employing power utility functions, where increased risk aversion implies a higher discount rate implies a lower CO_2 price

2. CO_2 price *declines* over time

   in contrast to most standard models with the exception of Ulph & Ulph (1994) [producer behavior], Acemoglu et al (2012) [shift from “dirty” to “clean”], Lemoine & Rudik (2017) [inertia]

3. Increased risk aversion increases risk premium relative to expected damages

   in contrast to standard models due to their use of power utility functions and (typically) lack of possibility for ‘catastrophic’ damages

4. Enormous social costs of delay

   in contrast to most standard models, which often estimate cost of delay based on (rising) ‘optimal’ CO_2 price over time in any given year (e.g. Nordhaus 2017, Changes in the DICE model, 1992 – 2017)

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

3 We decompose CO_2 price into two components

Optimal CO_2 price = expected damages + risk premium

CO_2 price reflects future state-dependent damages, $D_{s,t}$, weighted by their probability, $\pi_{s,t}$, and pricing kernel $m_{s,t} = \left(\frac{\partial U}{\partial c_{s,t}}\right) / \left(\frac{\partial U}{\partial c_0}\right)$:

$$\sum_{t=1}^{T} \sum_{s=1}^{S(t)} \pi_{s,t} m_{s,t} D_{s,t} \left(= \sum_{t=1}^{T} E_0\left[\tilde{m}_t \tilde{D}_t\right] \right)$$

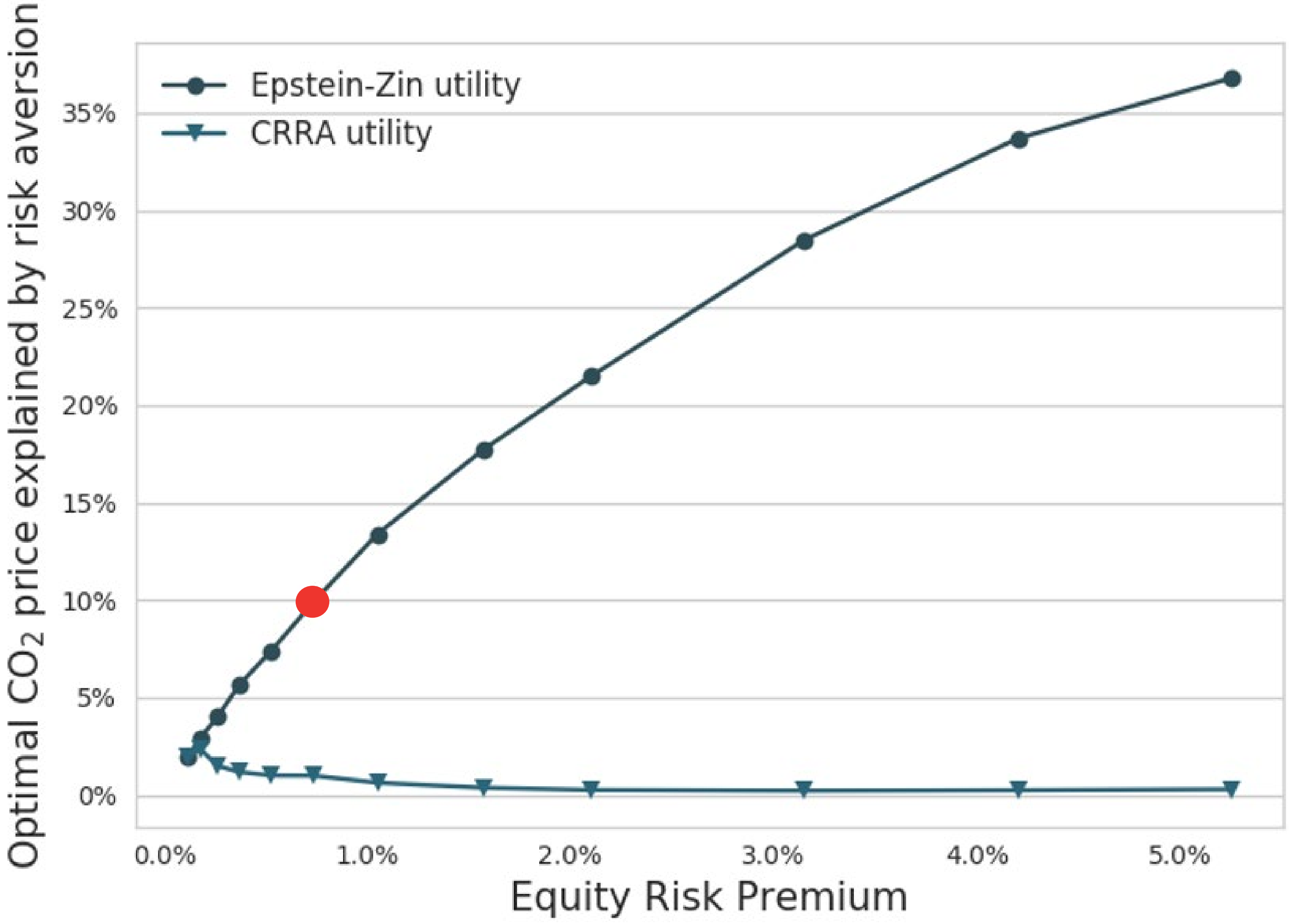
which we rearrange as:

$$\underbrace{\sum_{t=1}^{T} E_0[\tilde{m}_t] \cdot E_0\left[\tilde{D}_t\right]}_{Expected\ Damages} + \underbrace{\sum_{t=1}^{T} cov_0\left(\tilde{m}_t, \tilde{D}_t\right)}_{Risk\ Premium}$$

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

3 Epstein-Zin utility allows risk premium to play a significant role

Increased risk aversion increases risk premium relative to expected damages



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Four novel conclusions:

1. Increased risk aversion *increases* the CO_2 price

   in contrast to most standard models employing power utility functions, where increased risk aversion implies a higher discount rate implies a lower CO_2 price

2. CO_2 price *declines* over time

   in contrast to most standard models with the exception of Ulph & Ulph (1994) [producer behavior], Acemoglu et al (2012) [shift from “dirty” to “clean”], Lemoine & Rudik (2017) [inertia]

3. Increased risk aversion increases risk premium relative to expected damages

   in contrast to standard models due to their use of power utility functions and (typically) lack of possibility for ‘catastrophic’ damages

4. Enormous social costs of delay

   in contrast to most standard models, which often estimate cost of delay based on (rising) ‘optimal’ CO_2 price over time in any given year (e.g. Nordhaus 2017, Changes in the DICE model, 1992 – 2017)

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

4 Enormous social costs of delay

Cost of delay increases roughly with the square of time

Q: How much additional consumption is required throughout the first period to bring the utility with first-period mitigation set to zero up to the unconstrained level?

First-period length	*Annual* consumption impact during first period	*Annual / Total* lump-sum compensation estimate
5 years	11%	~$5 trillion / ~$24 trillion
10 years	23%	~$10 trillion / ~ $100 trillion
15 years	36%	~$15 trillion / ~$230 trillion

Each year of delay causes the equivalent consumption loss *over the entire first period* to increase by roughly 2.3%

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Four novel conclusions:

1. Increased risk aversion *increases* the CO_2 price

   in contrast to most standard models employing power utility functions, where increased risk aversion implies a higher discount rate implies a lower CO_2 price

2. CO_2 price *declines* over time

   in contrast to most standard models with the exception of Ulph & Ulph (1994) [producer behavior], Acemoglu et al (2012) [shift from “dirty” to “clean”], Lemoine & Rudik (2017) [inertia]

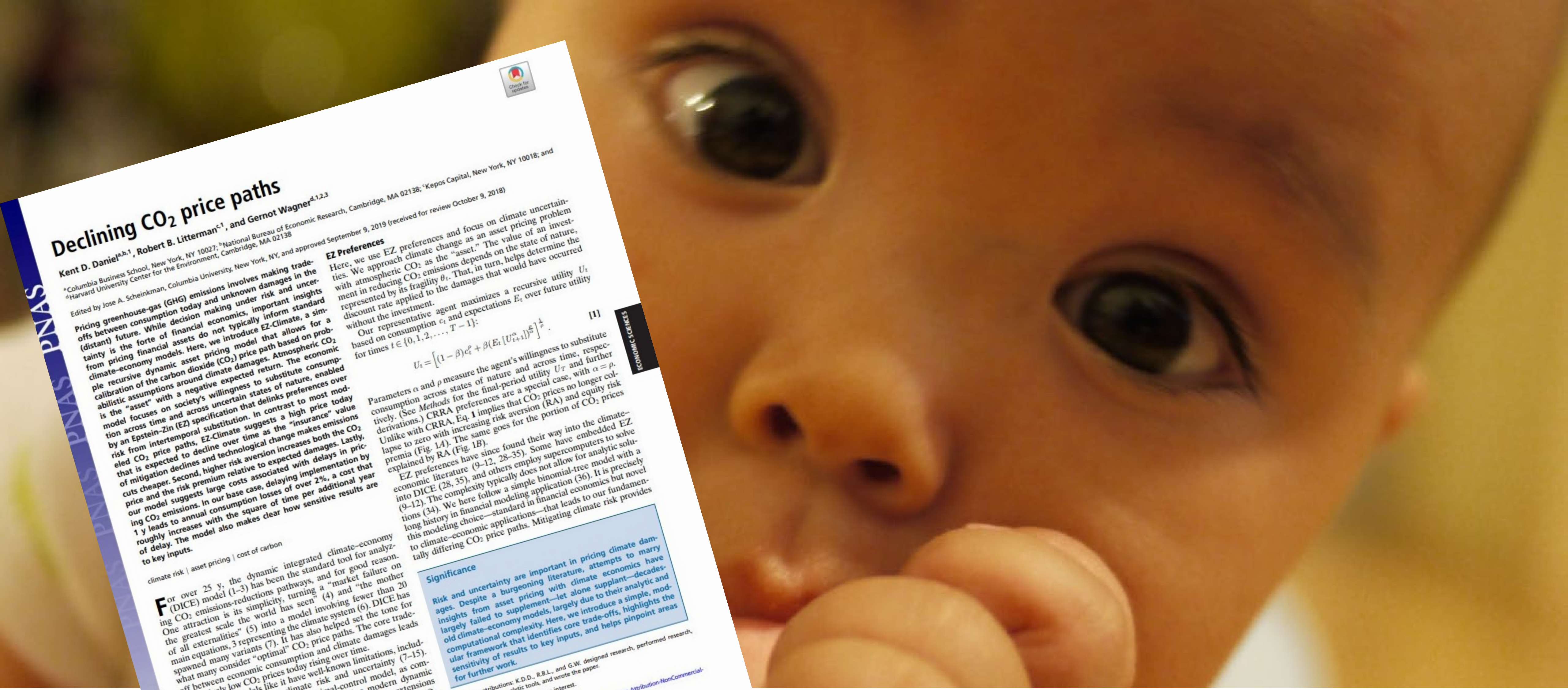
3. Increased risk aversion increases risk premium relative to expected damages

   in contrast to standard models due to their use of power utility functions and (typically) lack of possibility for ‘catastrophic’ damages

4. Enormous social costs of delay

   in contrast to most standard models, which often estimate cost of delay based on (rising) ‘optimal’ CO_2 price over time in any given year (e.g. Nordhaus 2017, Changes in the DICE model, 1992 – 2017)

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate



Check for updates

Declining CO_2 price paths

Kent D. Daniel[a,b,1], Robert B. Litterman[c,1], and Gernot Wagner[d,1,2,3]

[a]Columbia Business School, New York, NY 10027; [b]National Bureau of Economic Research, Cambridge, MA 02138; [c]Kepos Capital, New York, NY 10018; and [d]Harvard University Center for the Environment, Cambridge, MA 02138

Edited by Jose A. Scheinkman, Columbia University, New York, NY, and approved September 9, 2019 (received for review October 9, 2018)

Pricing greenhouse-gas (GHG) emissions involves making trade-offs between consumption today and unknown damages in the (distant) future. While decision making under risk and uncertainty is the forte of financial economics, important insights from pricing financial assets do not typically inform standard climate–economy models. Here, we introduce EZ-Climate, a simple recursive dynamic asset pricing model that allows for probabilistic calibration of the carbon dioxide (CO_2) price path based on probabilistic assumptions around climate damages. Atmospheric CO_2 is the "asset" with a negative expected return. The economic model focuses on society's willingness to substitute consumption across time and across uncertain states of nature, enabled by an Epstein-Zin (EZ) specification that delinks preferences over risk from intertemporal substitution. In contrast to most modeled CO_2 price paths, EZ-Climate suggests a high price today that is expected to decline over time as the "insurance" value of mitigation declines and technological change makes emissions cuts cheaper. Second, higher risk aversion increases both the CO_2 price and the risk premium relative to expected damages. Lastly, our model suggests large costs associated with delays in pricing CO_2 emissions. In our base case, delaying implementation by 1 y leads to annual consumption losses of over 2%, a cost that roughly increases with the square of time per additional year of delay. The model also makes clear how sensitive results are to key inputs.

climate risk | asset pricing | cost of carbon

For over 25 y, the dynamic integrated climate–economy (DICE) model (1–3) has been the standard tool for analyzing CO_2 emissions-reductions pathways, and for good reason. One attraction is its simplicity, turning a "market failure on the greatest scale the world has seen" (4) and "the mother of all externalities" (5) into a model involving fewer than 20 main equations, 3 representing the climate system (6). DICE has spawned many variants (7). It has also helped set the tone for what many consider "optimal" CO_2 price paths. The core trade-off between economic consumption and climate damages leads to relatively low CO_2 prices today rising over time.

DICE and models like it have well-known limitations, including how they represent climate risk and uncertainty (7–15). DICE, for example, is not an optimal-control model, as commonly understood by economists employing modern dynamic economic analysis, even though it lends itself to those extensions (9–12). The underlying structure all but prescribes a rising CO_2 price path over time.

One important limitation is the form of the utility function. Constant relative risk aversion (CRRA) preferences, standard in most climate–economy models (1, 7, 16), assume that economic agents have an equal aversion to variation in consumption across states of nature and over time. Evidence from financial markets suggests that this is not the case (17). The risk premium (RP) of equities over bonds points to a fundamental difference in how much society is willing to pay to substitute consumption risk across states of nature compared to over time (18, 19). Some have explained the discrepancy by allowing for extreme events (20–22), and others have looked to more flexible preferences (23–26) or both (27). Our own preference specification follows Zin (EZ) (24, 25).

EZ Preferences

Here, we use EZ preferences and focus on climate uncertainties. We approach climate change as an asset pricing problem with atmospheric CO_2 as the "asset." The value of an investment in reducing CO_2 emissions depends on the state of nature, represented by its fragility θ_t. That, in turn, helps determine the discount rate applied to the damages that would have occurred without the investment.

Our representative agent maximizes a recursive utility U_t based on consumption c_t and expectations E_t over future utility for times $t \in \{0, 1, 2, \ldots, T-1\}$:

$$U_t = \left[(1-\beta)c_t^{\rho} + \beta\left(E_t\left[U_{t+1}^{\alpha}\right]\right)^{\frac{\rho}{\alpha}}\right]^{\frac{1}{\rho}}. \quad [1]$$

Parameters α and ρ measure the agent's willingness to substitute consumption across states of nature and across time, respectively. (See *Methods* for the final-period utility U_T and further derivations.) CRRA preferences are a special case, with $\alpha = \rho$. Unlike with CRRA, Eq. **1** implies that CO_2 prices no longer collapse to zero with increasing risk aversion (RA) and equity risk premia (Fig. 1*A*). The same goes for the portion of CO_2 prices explained by RA (Fig. 1*B*).

EZ preferences have since found their way into the climate–economic literature (9–12, 28–35). Some have embedded EZ into DICE (28, 35), and others employ supercomputers to solve (9–12). The complexity typically does not allow for analytic solutions (34). We here follow a simple binomial-tree model with a long history in financial modeling application (36). It is precisely this modeling choice—standard in financial economics but novel to climate–economic applications—that leads to our fundamentally differing CO_2 price paths. Mitigating climate risk provides

Significance

Risk and uncertainty are important in pricing climate damages. Despite a burgeoning literature, attempts to marry insights from asset pricing with climate economics have largely failed to supplement—let alone supplant—decades-old climate–economy models, largely due to their analytic and computational complexity. Here, we introduce a simple, modular framework that identifies core trade-offs, highlights the sensitivity of results to key inputs, and helps pinpoint areas for further work.

Author contributions: K.D.D., R.B.L., and G.W. designed research, performed research, contributed new analytic tools, and wrote the paper.

The authors declare no competing interest.

This article is a PNAS Direct Submission.

[1]K.D.D., R.B.L., and G.W. contributed equally to this work.

[2]Present addresses: Department of Environmental Studies, New York University, New York, NY 10003; and Robert F. Wagner Graduate School of Public Service, New York University, New York, NY 10012.

[3]To whom correspondence may be addressed. Email: gernot@gwagner.com

This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1817444116/-/DCSupplemental.

ECONOMIC SCIENCES

Gernot Wagner
gwagner@nyu.edu
gwagner.com/ezclimate

Background

Climate change as a standard asset pricing problem

CO_2 in the atmosphere as an asset, albeit with negative payoffs

- Model based on Summers & Zeckhauser (2008) to capture climate change risk and uncertainty

- Epstein-Zin (1989, 1991)-Weil (1990) preferences to allow separation of intertemporal marginal rate of substitution and risk aversion:

$$U_t = \left[(1-\beta)c_t^{\rho} + \beta\left[(E_t[U_{t+1}^{\alpha}])^{1/\alpha}\right]^{\rho}\right]^{1/\rho}$$

  ρ measures agent's willingness to substitute across time

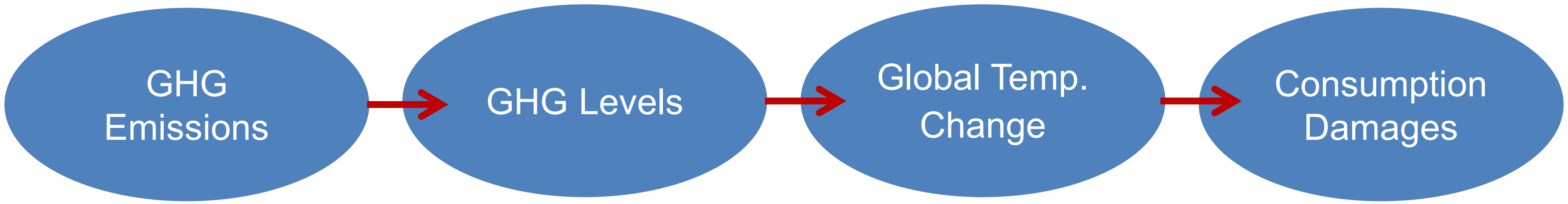
  α measures agent's willingness to substitute across states of nature

- A *simple*, six-period, recombining tree structure solved numerically

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

The basic model

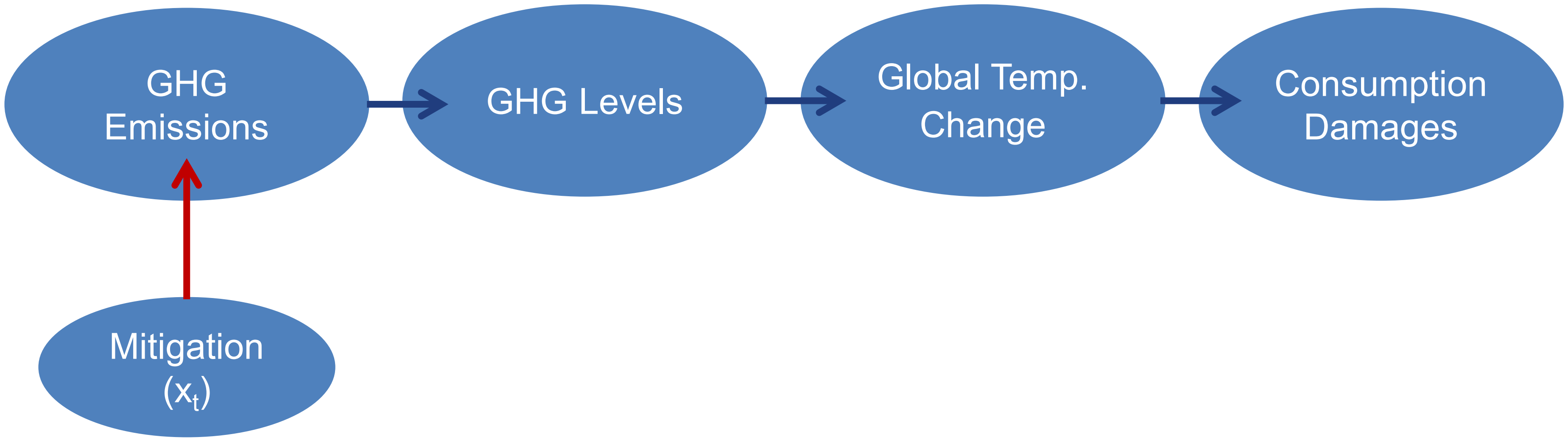
Greenhouse gas emissions, and their mitigation, affect damage outcomes



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

The basic model

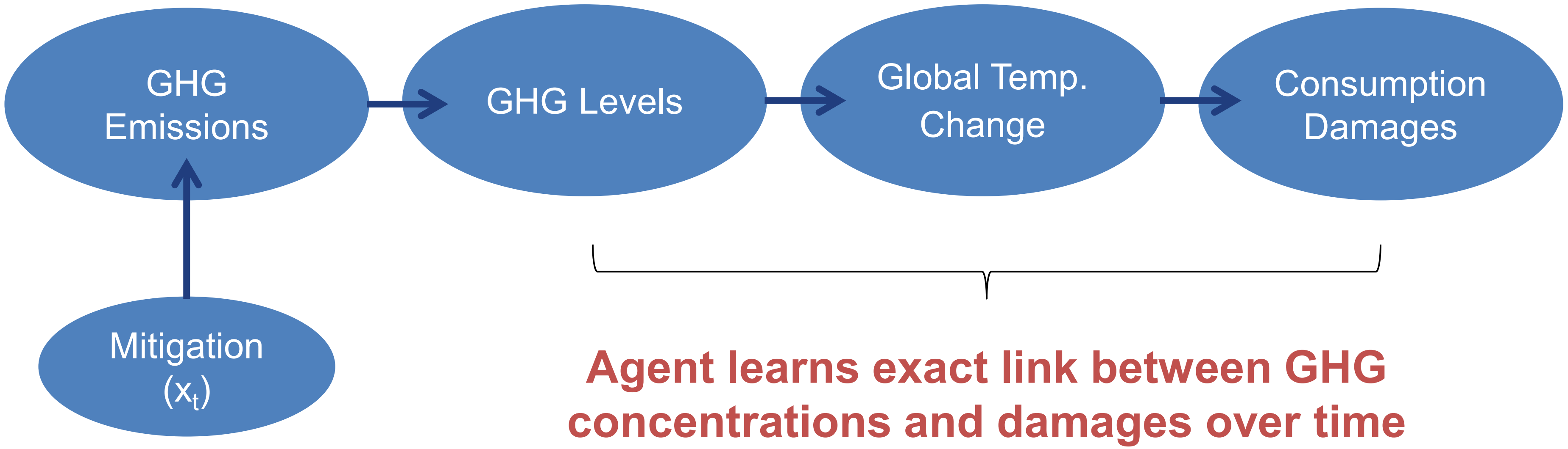
Greenhouse gas emissions, and their mitigation, affect damage outcomes



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

The basic model

Uncertain relationship between greenhouse gas levels and consumption damages

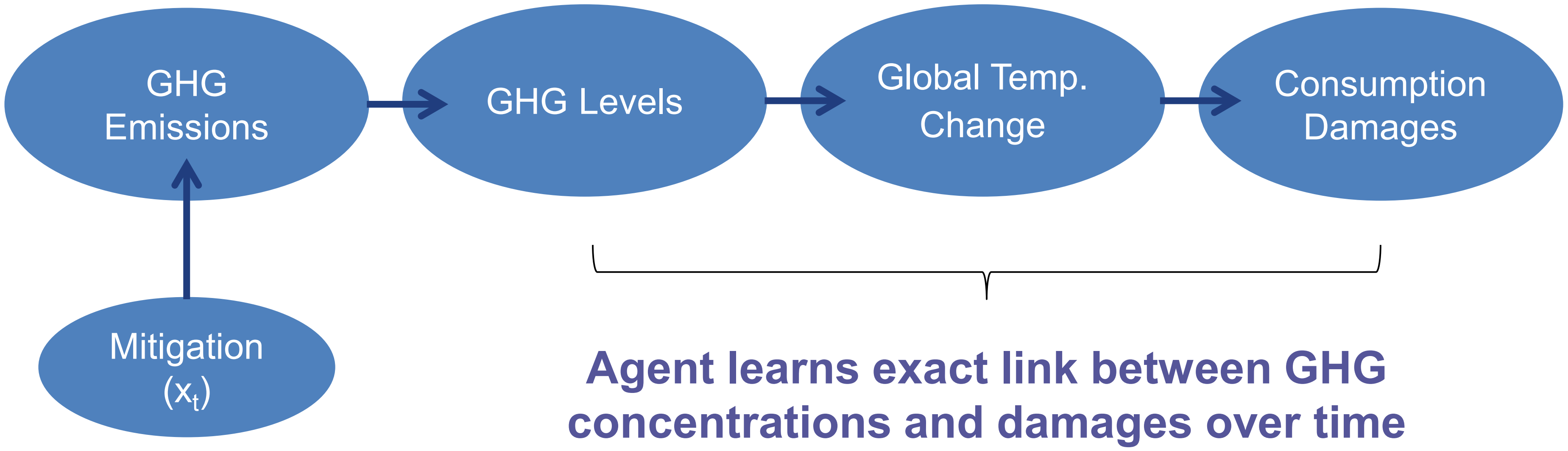


Agent learns exact link between GHG concentrations and damages over time

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

The basic model

Uncertain relationship between greenhouse gas levels and consumption damages

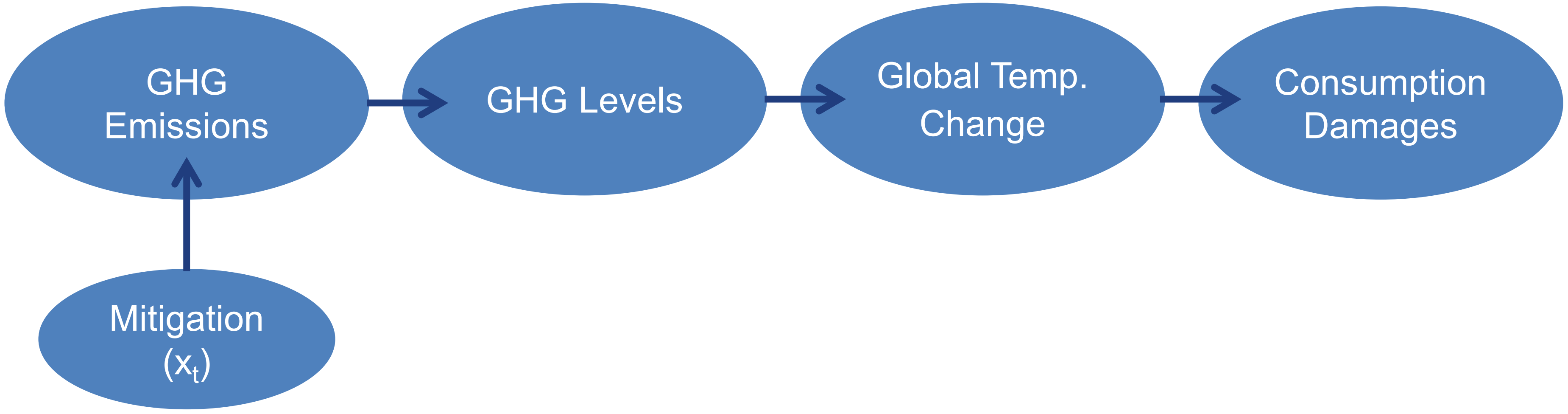


Higher risk aversion, higher mitigation

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

The basic model

Consumption as a function of future climate damages



- Discrete time, T + 1 periods
- Base case: agent's "endowed" consumption $\bar{C}_t$ grows at 1.5%/year
- Agent's actual consumption:

$$C_t = \bar{C}_t \cdot \left(1 - D_t(X_t, \theta_t) - \kappa_t(x_t)\right)$$

where $D_t(X_t, \theta_t)$ = damage, fractional to consumption

X_t = total mitigation through time *t*

Solve for $x_t{}^*$

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Calibrated cost function

Incorporating technological change into the cost function for emissions mitigation

$$C_t = \bar{C}_t \cdot (1 - D_t(CRF_t, \theta_t))\,\big(1 - \kappa_t(x_t)\big)$$

Allow for technological change of the form:

$$\kappa(x,t) = \kappa(x)[1 - \varphi_0 - \varphi_1 X(t)]^t$$

where X_t : average mitigation up to time t

φ_0 : exogenous technological change

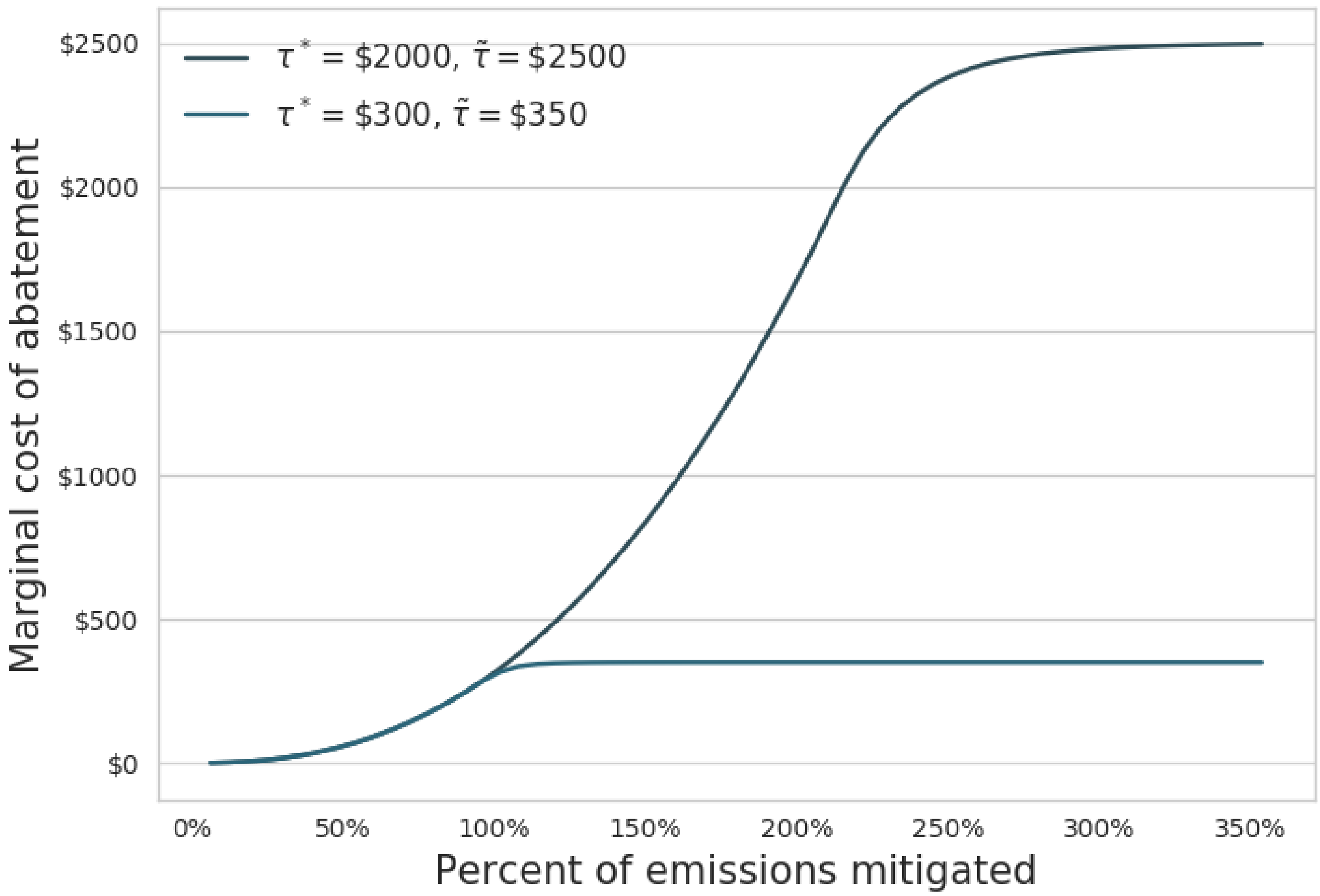
φ_1 : endogenous technological change

Note: if φ_1 > 0, a higher level of past mitigation leads to lower cost today

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Calibrated cost of mitigation in base case and with assumed backstop

Non-NPV-positive portion of McKinsey (2009), scaled to 2015 and fit to a power function



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Damage function

Damage is a function of GHG mitigation and the uncertain link from GHGs to final damages

$$C_t = \bar{C}_t \cdot (1 - D_t(CRF_t, \theta_t)) \left(1 - \kappa_t(x_t)\right)$$

Endowed consumption is reduced each period by (uncertain) multiplicative Consumption Damage factor:

$$(1 - D_t(CRF_t, \theta_t))$$

where CRF_t: Cumulative Radiative Forcing up until time t

θ_t : characterizes relation between GHGs and damages

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Damage function components

The damage function is made up of catastrophic and non-catastrophic components

$$D_t = \underbrace{(1 - L(\Delta T(t))}_{\text{non-catastrophic}} \cdot \underbrace{\left(1 - I_{TP}\left[1 - e^{-TP_{\text{damage}}}\right]\right)}_{\text{catastrophic}}$$

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Damage function components

The damage function is made up of catastrophic and non-catastrophic components

$$\mathrm{D}_t = \underbrace{\left(1 - \mathrm{L}(\Delta T(t))\right)}_{\text{non-catastrophic}} \cdot \underbrace{\left(1 - \mathrm{I}_{TP}\left[1 - \mathrm{e}^{-\mathrm{TP}_{\mathrm{damage}}}\right]\right)}_{\text{catastrophic}}$$

- The *non-catastrophic* component captures anticipated losses due to temperature changes

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Damage function components

The damage function is made up of catastrophic and non-catastrophic components

$$D_t = \underbrace{(1 - L(\Delta T(t))}_{\text{non-catastrophic}} \cdot \underbrace{\left(1 - I_{TP}\left[1 - e^{-TP_{\text{damage}}}\right]\right)}_{\text{catastrophic}}$$

- The *non-catastrophic* component captures anticipated losses due to temperature changes
- The *catastrophic* component captures losses due to tail events – low probability, potentially high impact

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Non-catastrophic damage

Mapping from GHG levels, to temperature change, to expected damages

$$D_t = (1 - L(\Delta T(t)) \cdot \left(1 - I_{TP}\left[1 - e^{-TP_{\text{damage}}}\right]\right)$$

where $\Delta T_t(X_t)$: mapping from GHG concentrations to temperature change using log-normal distribution (Weitzman 2009)

$L(\Delta T_t(X_t))$: mapping from temperature change to damages using displaced gamma distribution (Pindyck 2012)

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Catastrophic damage

Captures the possibility of ‘tipping points’

$$D_t = (1 - L(\Delta T(t)) \cdot \left(1 - I_{TP}\left[1 - e^{-TP_{\text{damage}}}\right]\right)$$

In each period, a Poisson process, with a hazard rate based on ΔT_t governing whether a ‘tipping point’ is hit.

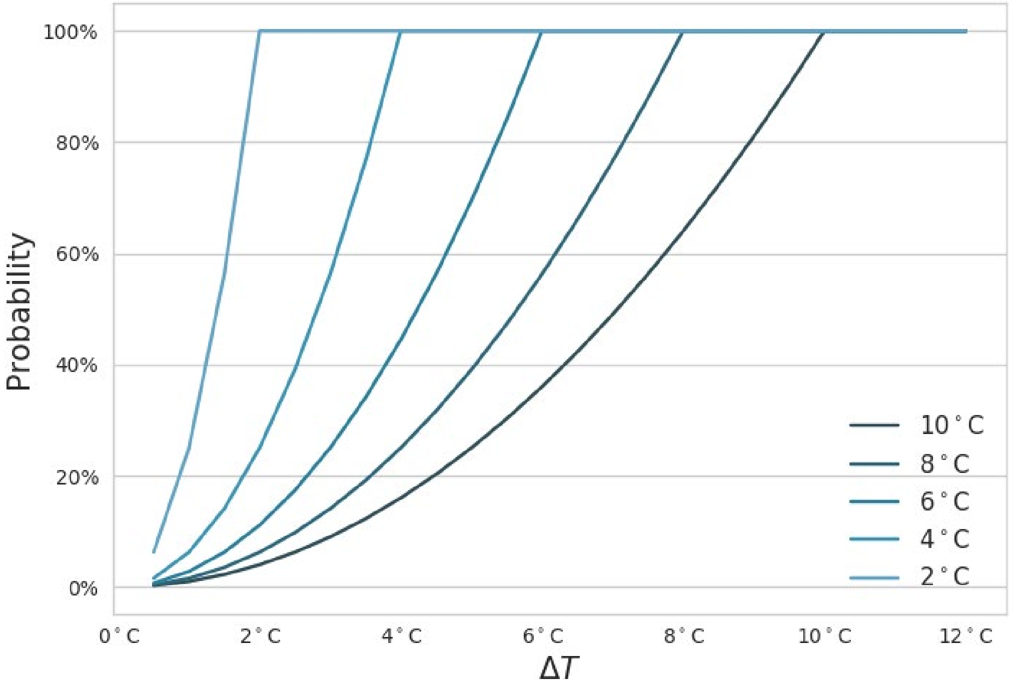
Once a tipping point is hit, the climate remains in a ‘catastrophic’ state through the final period, which results in additional fractional damage to consumption $e^{-TP_{\text{damage}}}$

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

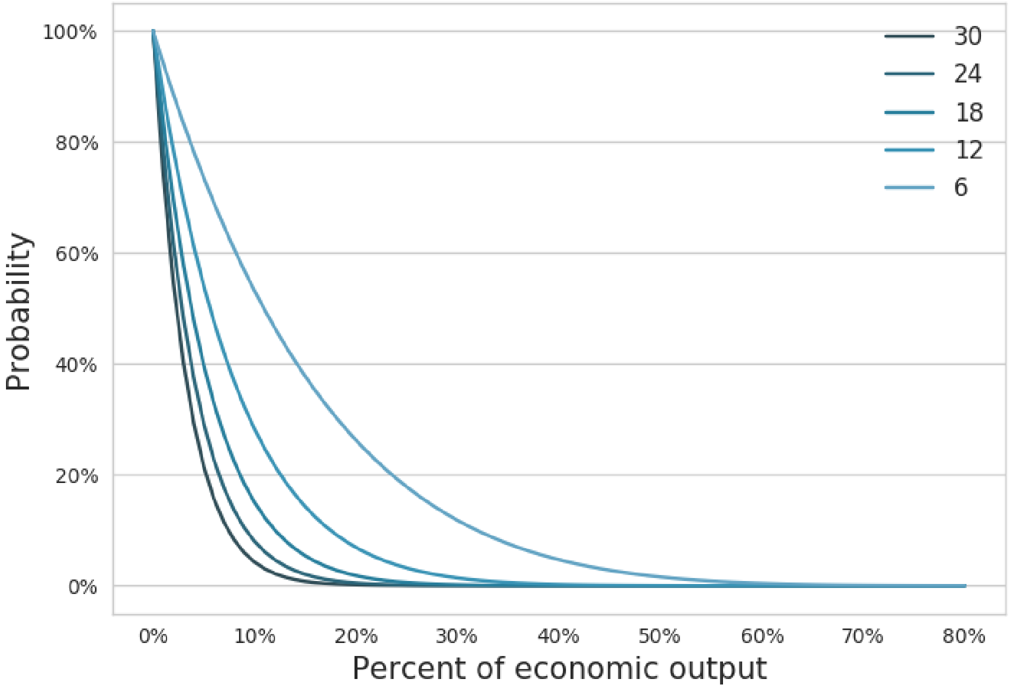
Tipping point probability and resulting damages crucial inputs

Scientific input needed for proper calibration

Probability of reaching a climatic tipping point as a function of *peakT*



Probability of damages greater than a particular percentage of output, given different levels of *disaster_tail*



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Solving the model…

Python code available via gwagner.com/ezclimate

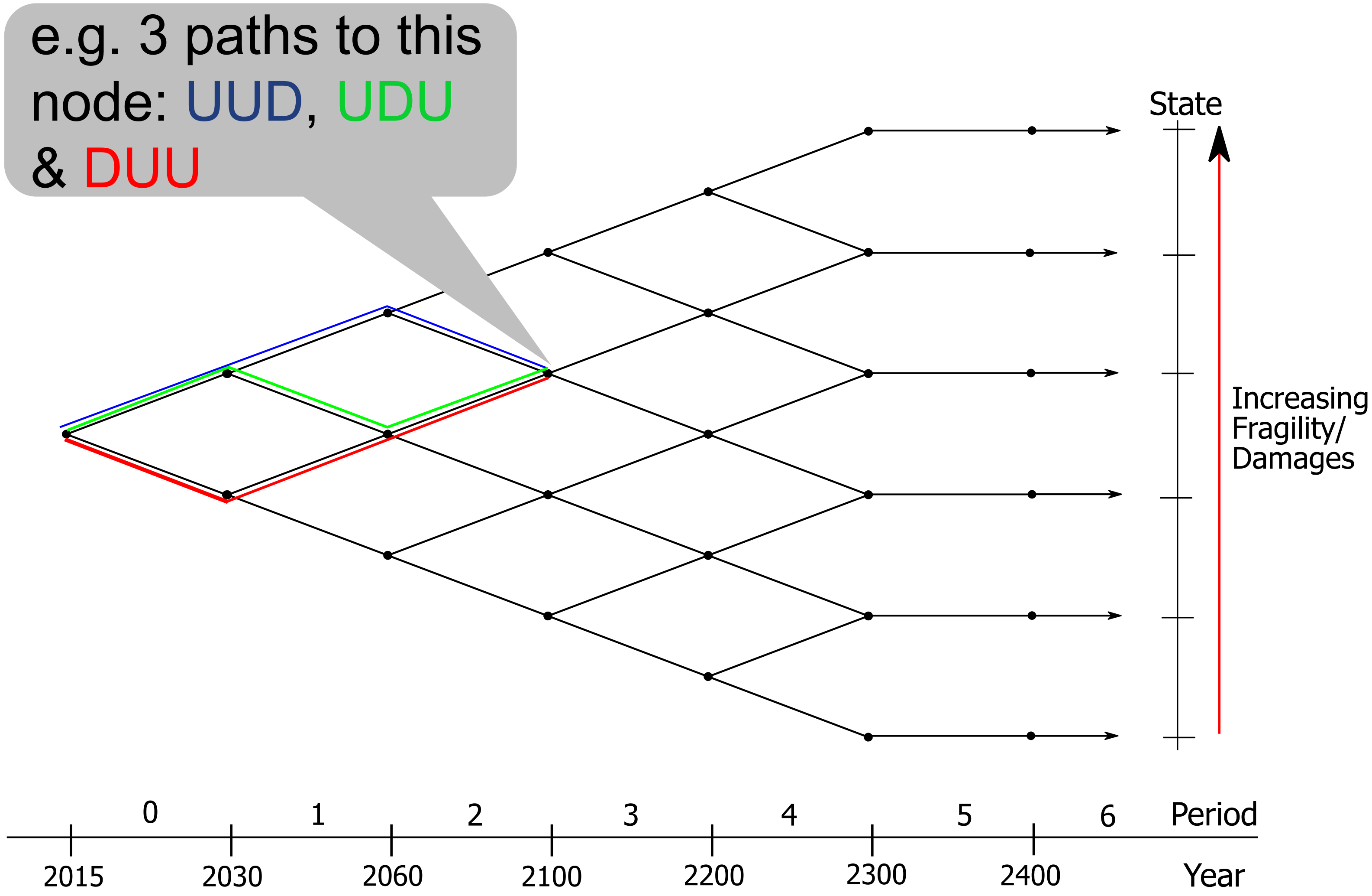
- 6 periods
- Recombining tree structure
- Ordered, equal probability states

- Numeric solution, selecting mitigation level x_t to maximize representative agent's expected utility
- Optimize for CO_2 price that implements this level of mitigation

Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

‘Recombining’ trees estimate outcome in each stage

Maximize representative agent’s utility, using Epstein-Zin preferences

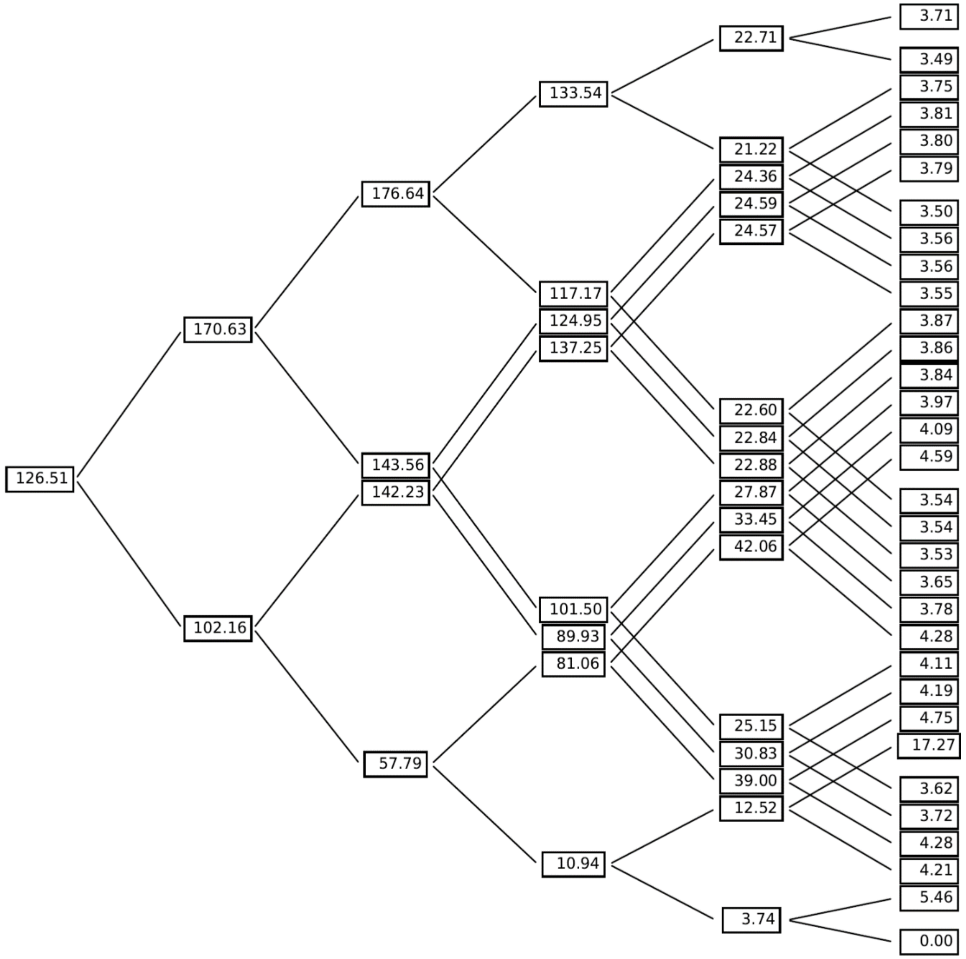


Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

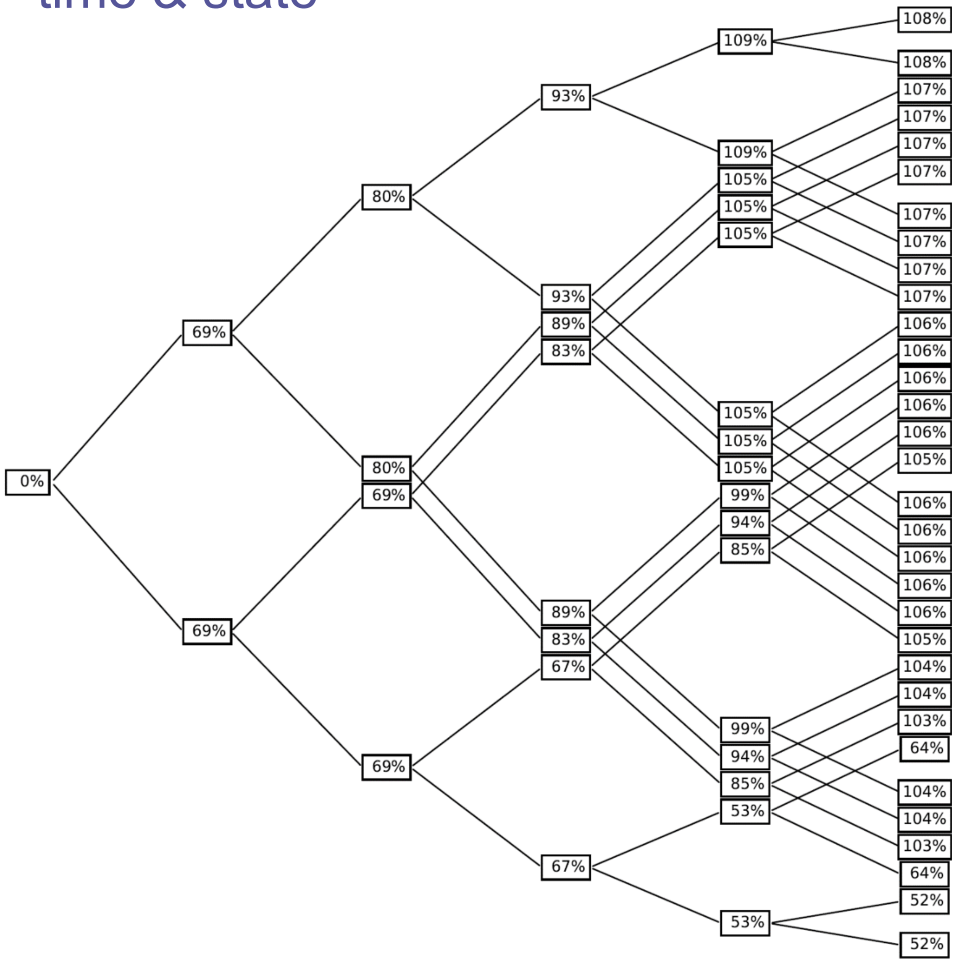
‘Recombining’ trees estimate outcome in each stage

Maximize representative agent’s utility, using Epstein-Zin preferences

Price per ton of CO_2 across time & states



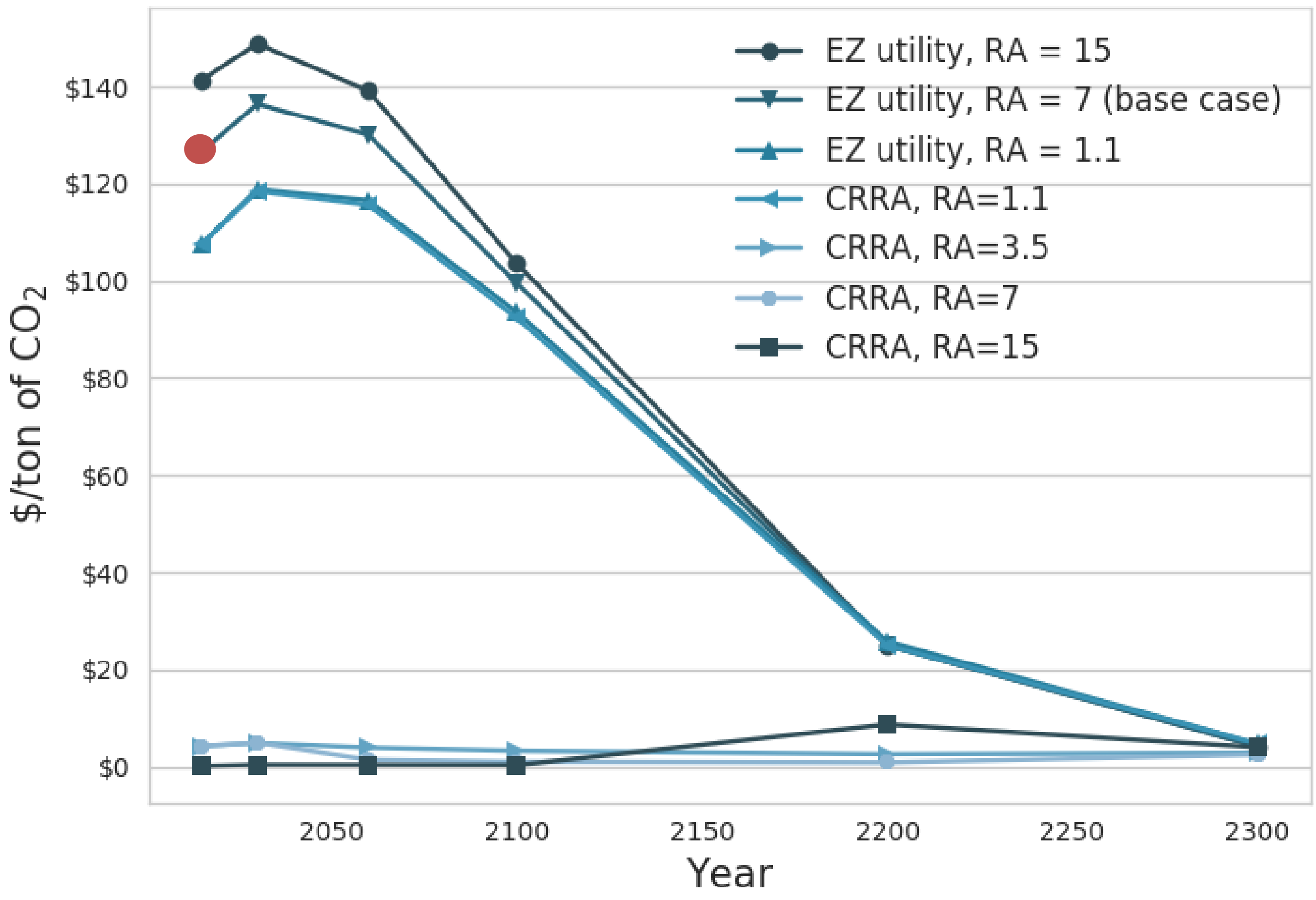
Average mitigation up to a particular time & state



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

CO_2 price sensitive to utility specification for 'reasonable' RA values

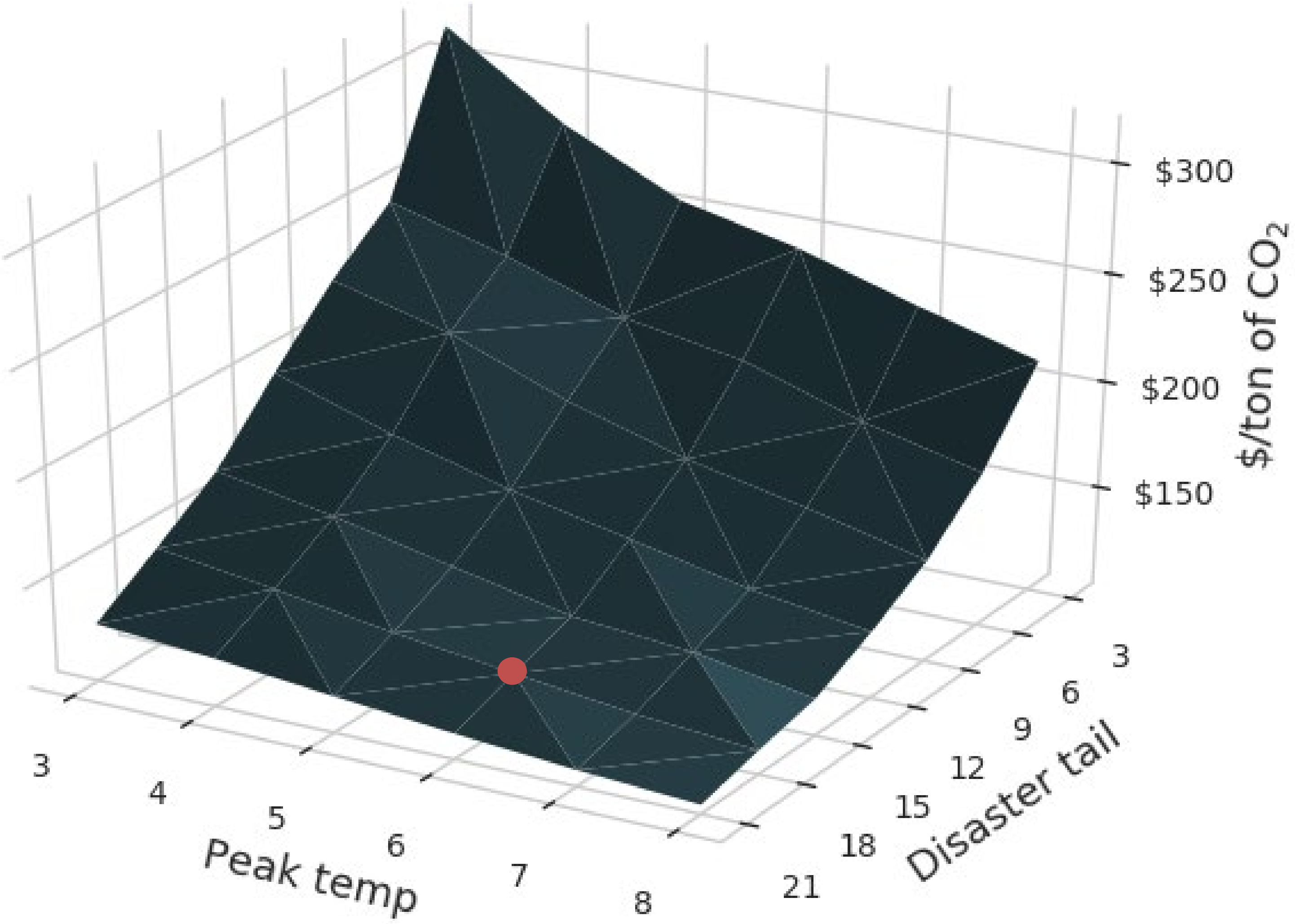
No difference between CRRA and EZ utility at RA=1.1, large differences for RA>~3



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

2015 CO_2 price increases with decreasing $peakT$ and $disaster\_tail$

Base case assumes *peakT* = 6 and *disaster_tail* = 18



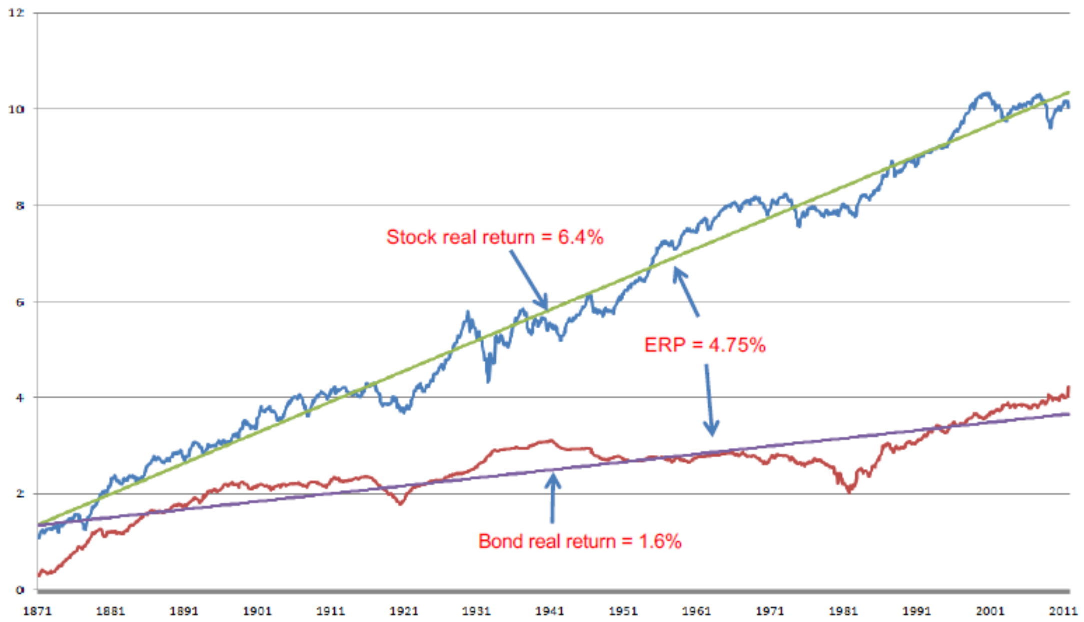
Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Further sensitivity analyses

1 Increased risk aversion *increases* CO_2 prices

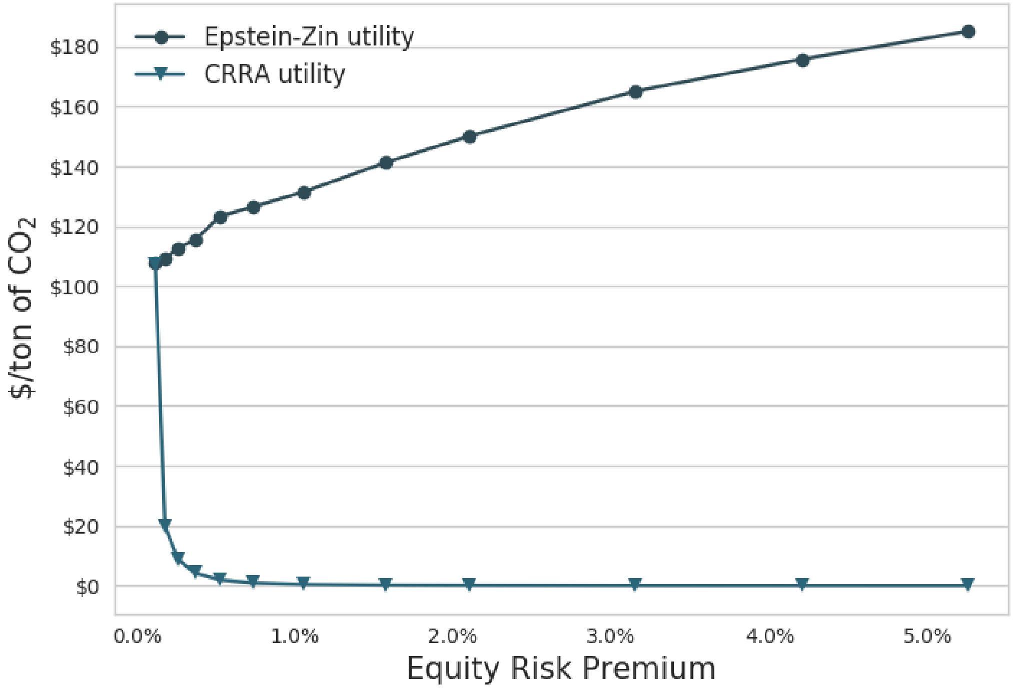
With CRRA utility, high risk aversion implies high discount rate implies lower CO_2 price

Log real return for stocks and bonds with fitted trend lines



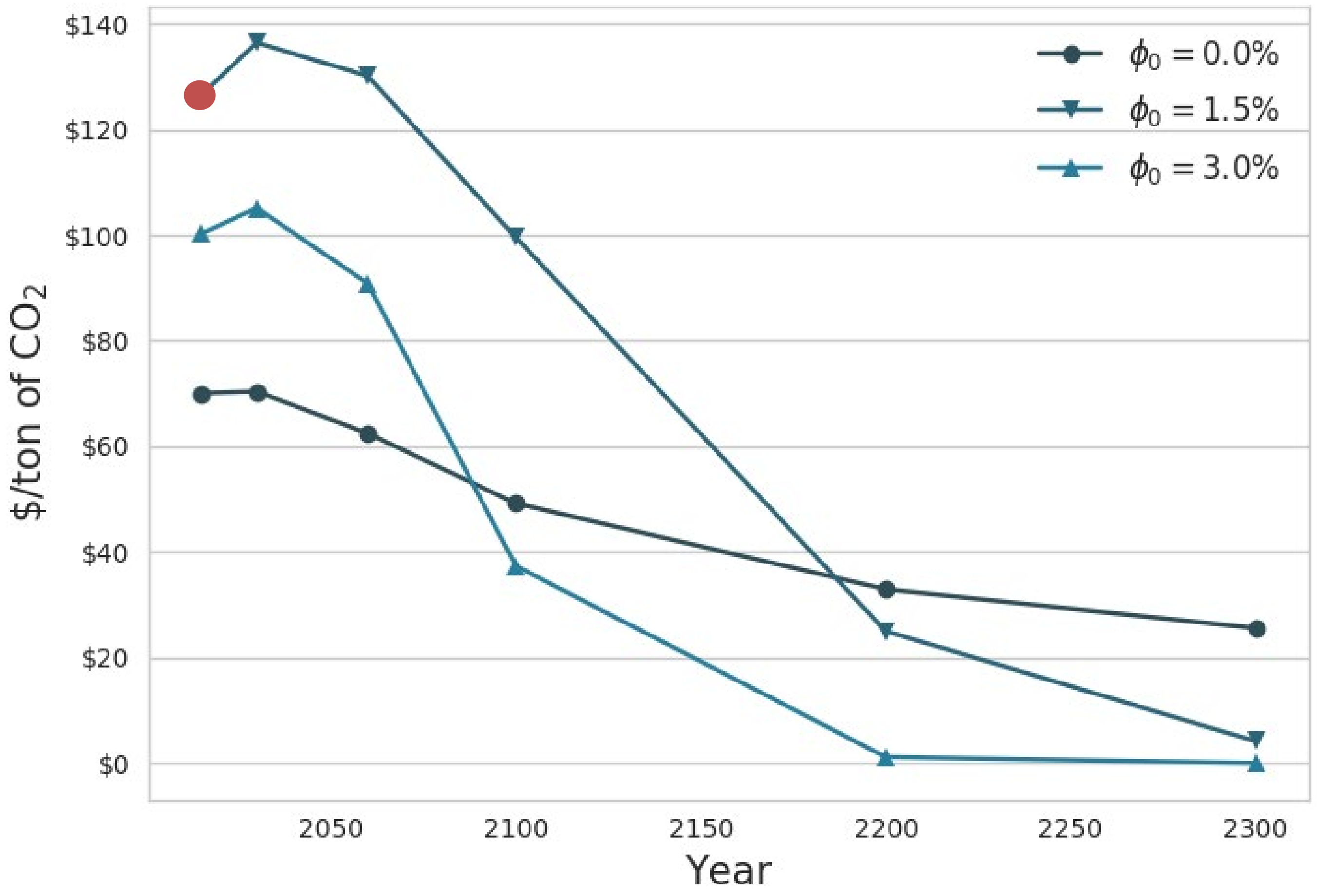
Source: Return data from Shiller (2000) and since continuously updated: http://www.econ.yale.edu/~shiller/data.htm

Epstein-Zin utility separates risk across time and states of nature



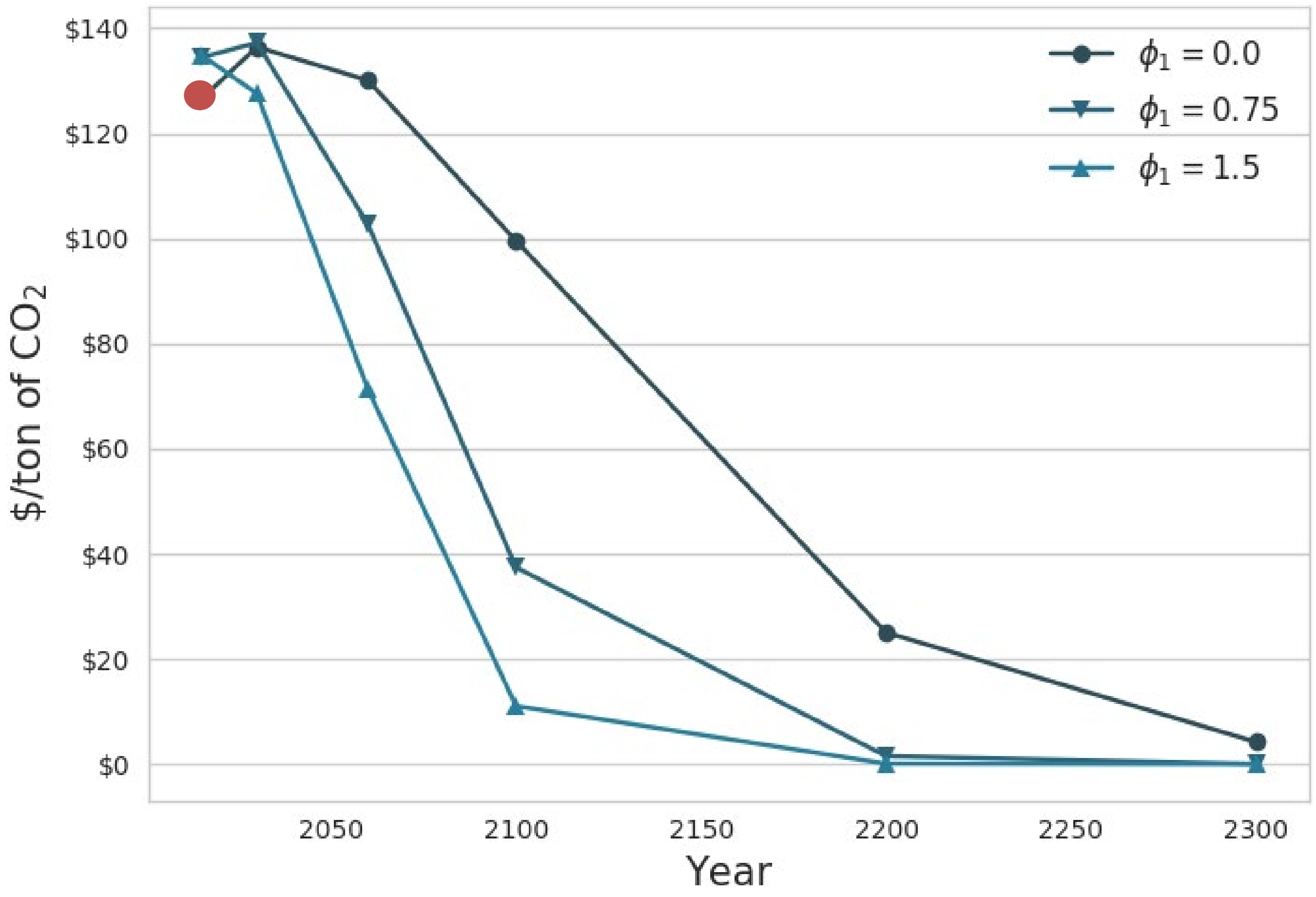
Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

CO_2 price in early years first increases then decreases with higher exogenous technical change, φ_0



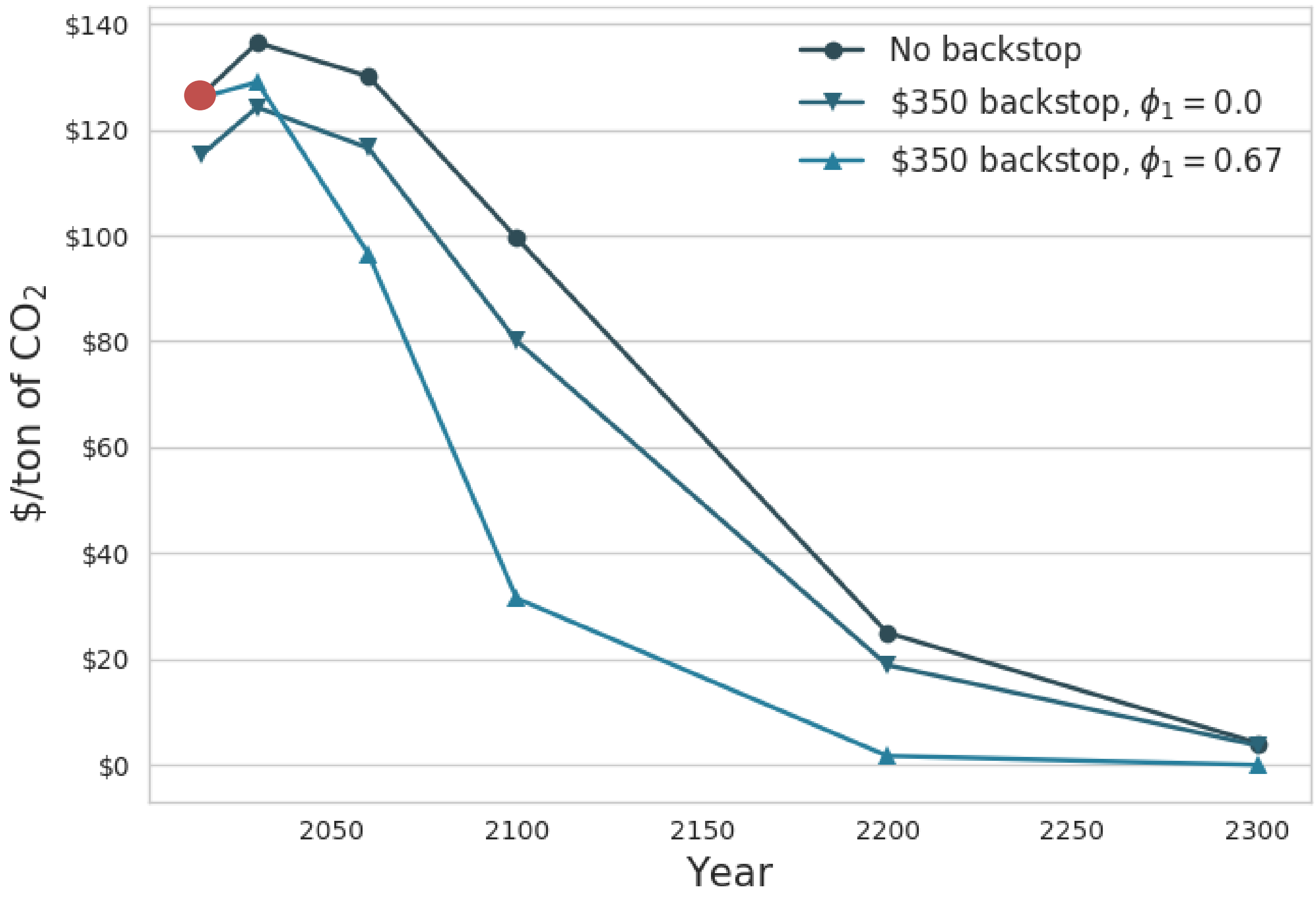
Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

CO_2 price decreases with increased endogenous technical change in later years



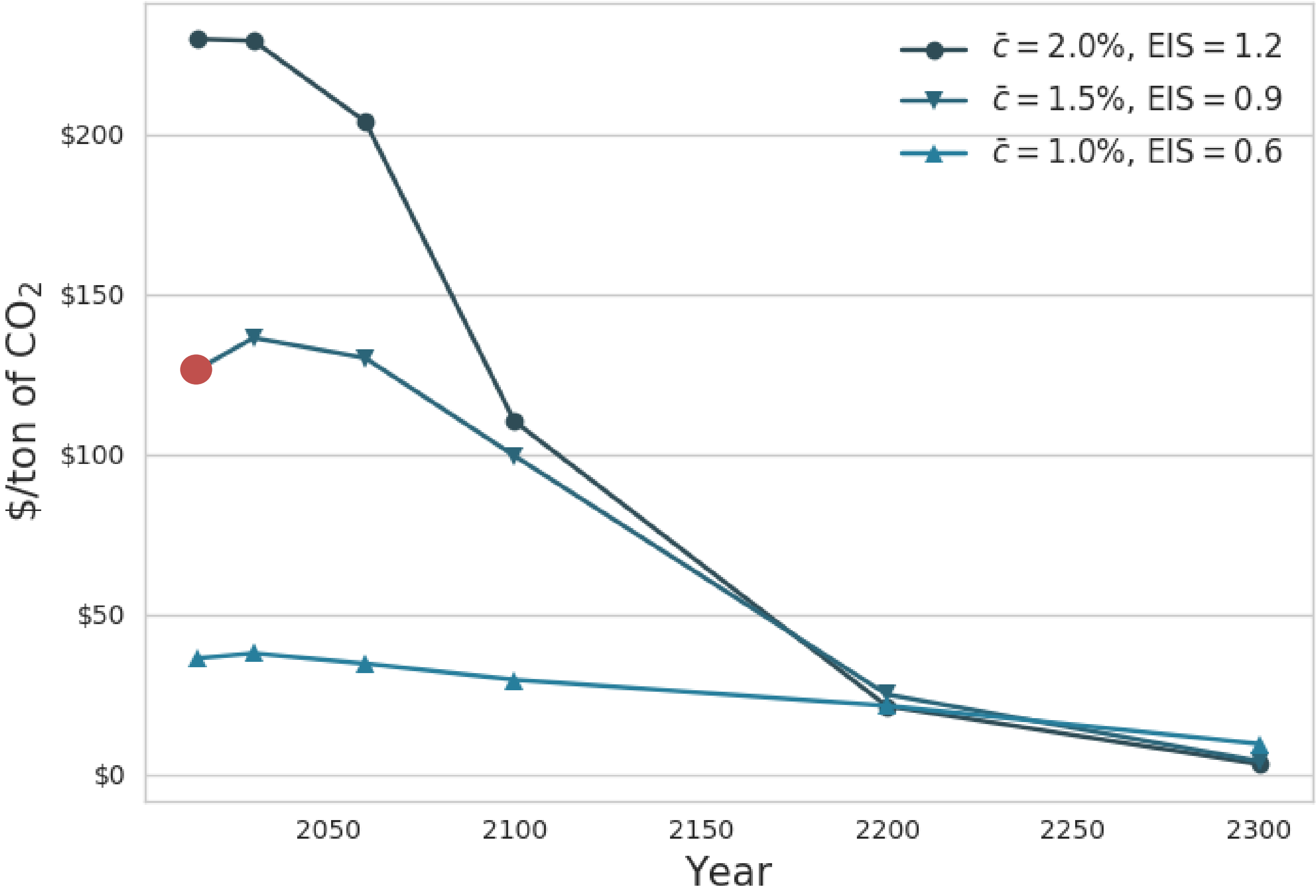
Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

CO_2 price decreases with backstop, with or without endogenous technological change



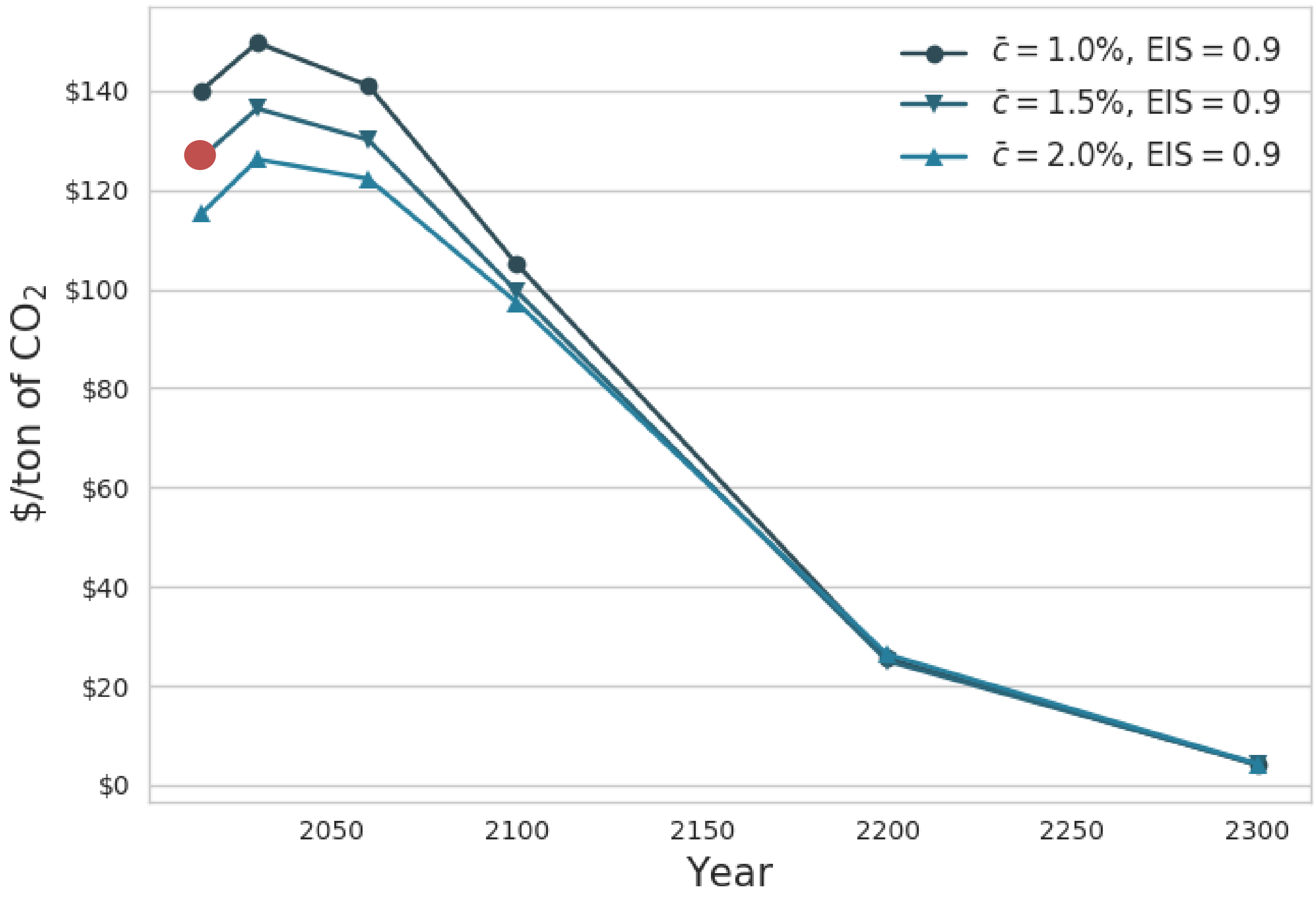
Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Increasing economic growth, while keep real interest rates constant, increases CO_2 prices dramatically in early periods



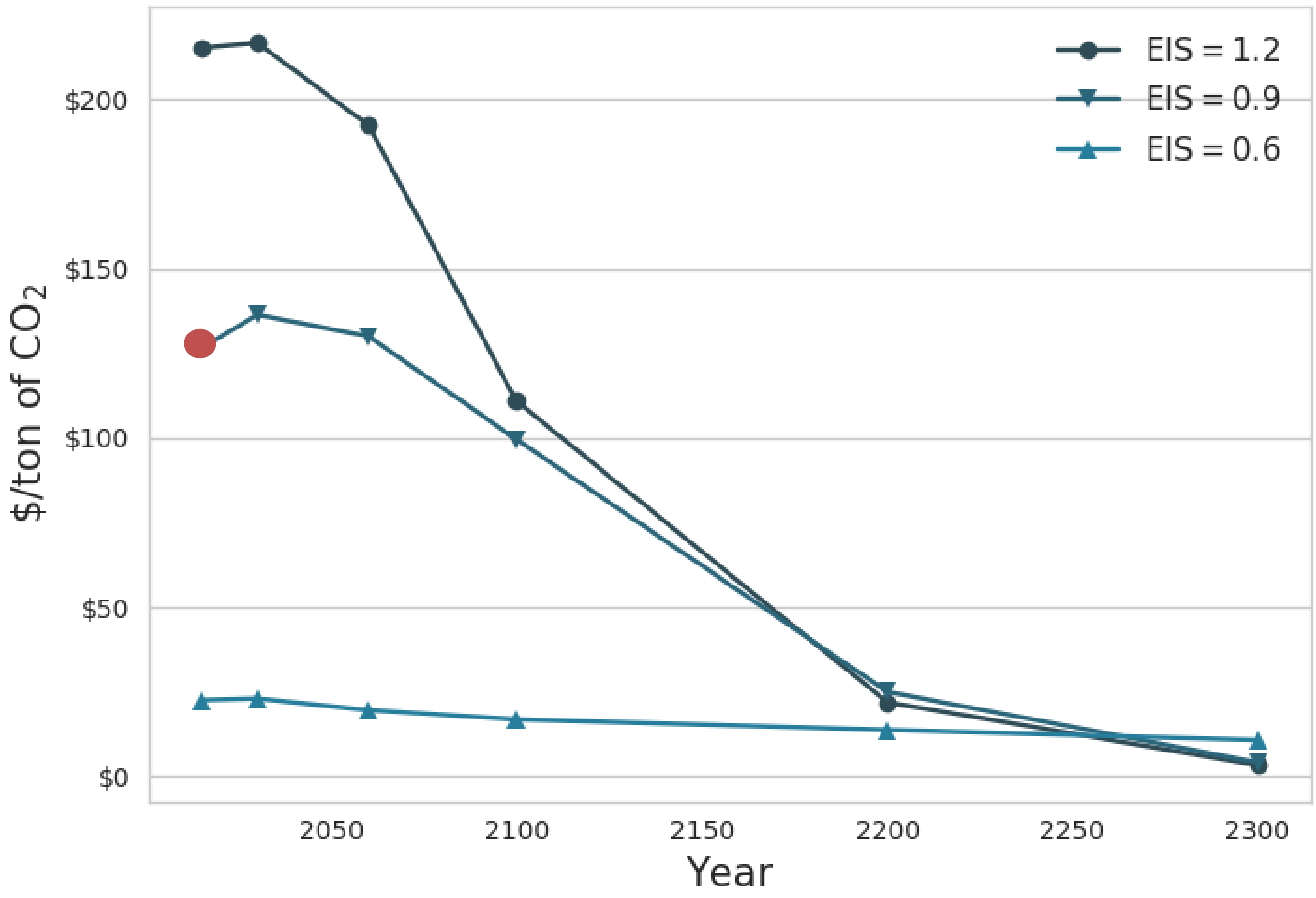
Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

Changing economic growth rates, while keeping EIS constant at 0.9, has little impact on CO_2 prices



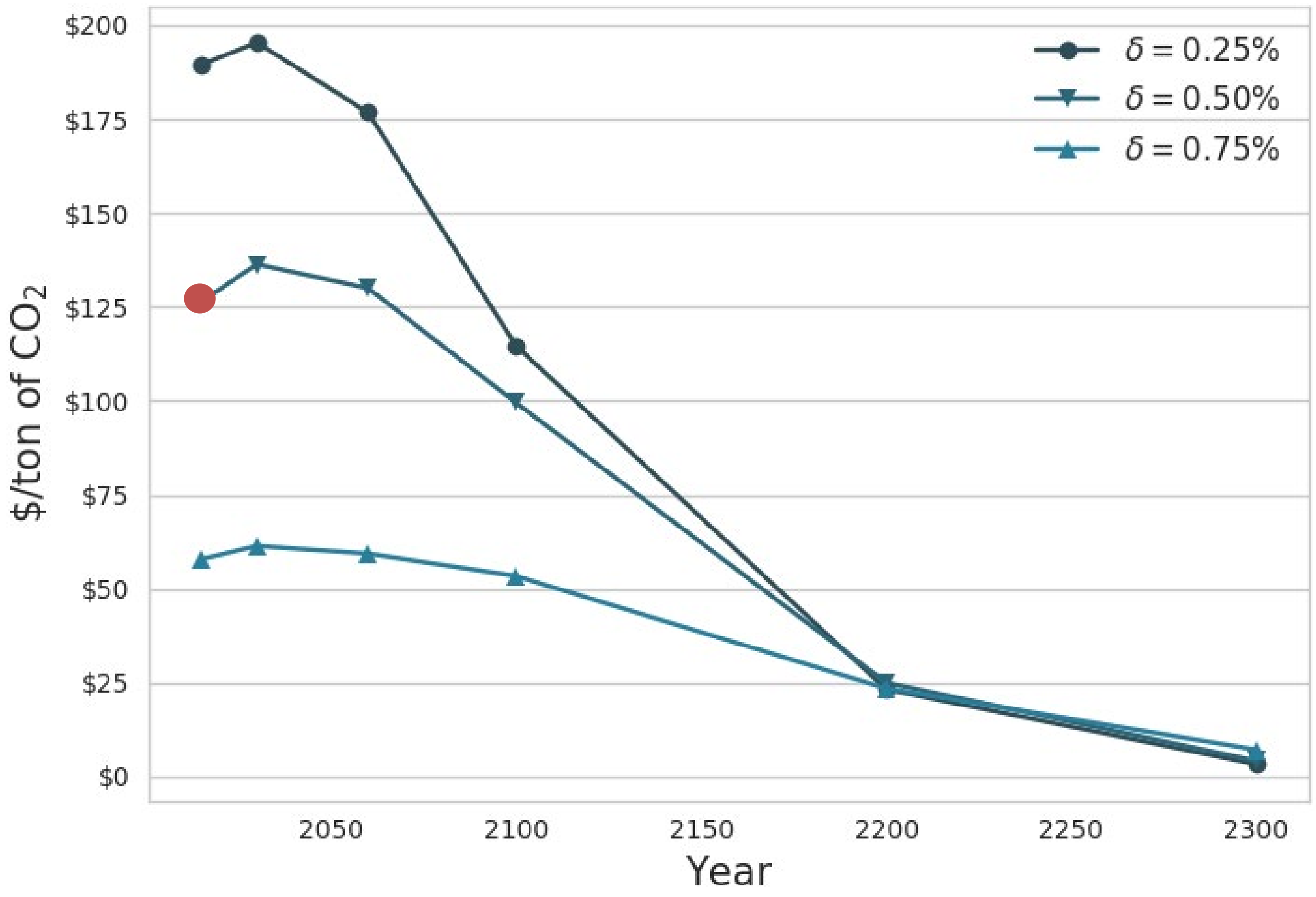
Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

A higher EIS goes hand-in-hand with a higher CO_2 price in early years



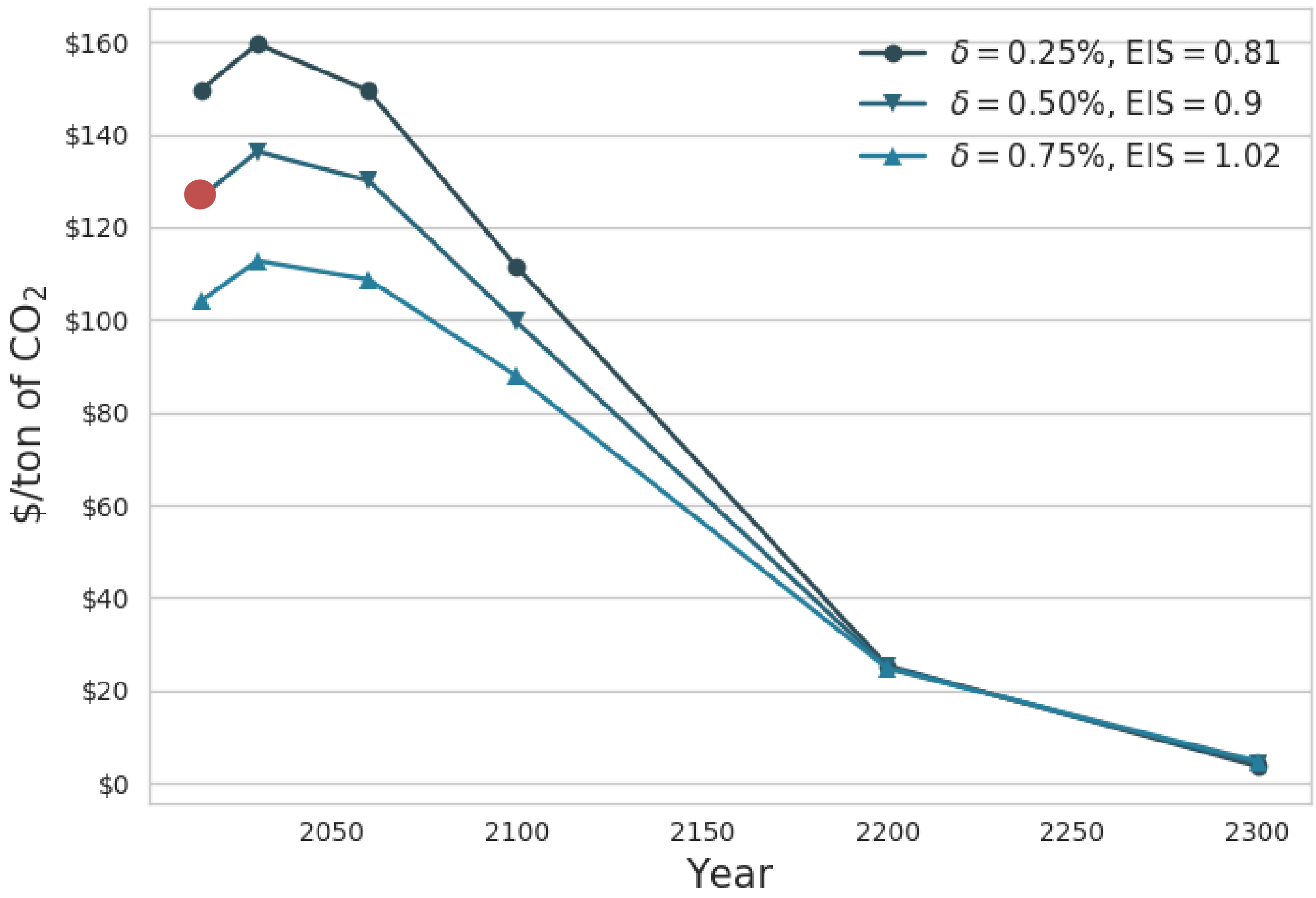
Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

CO_2 prices increase (in early years) with decreasing pure rate of time preference, δ, holding EIS fixed at 0.90



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate

CO_2 price increases with decreasing pure rate of time preference, δ, holding real interest rates fixed, while adjusting EIS accordingly



Source: Daniel, Litterman & Wagner (NBER October 2018, PNAS 2019), gwagner.com/ezclimate