An Economic Anatomy of Optimal Climate Policy

Juan B. Moreno-Cruz, Gernot Wagner and David W. Keith*

September 11, 2018

Abstract

We introduce geoengineering into an optimal control model of climate economics. Together with mitigation and adaptation, carbon and solar geoengineering span all possible climate policies. Their wildly different characteristics have important implications for policy. We show in the context of our model that: (i) whether emissions are positive or zero the optimal carbon tax always equals the marginal cost of carbon geoengineering; (ii) the introduction of either form of geoengineering leads to higher emissions yet lower temperatures; (iii) in a world with above-optimal cumulative emissions, mitigation alone is insufficient and only a complete set of instruments can minimize climate damages.

JEL: D90, O44, Q48, Q54, Q58

Keywords: climate change, climate policy; mitigation, adaptation, carbon geoengineering, carbon dioxide removal, solar geoengineering, solar radiation management.

---

*Moreno-Cruz: University of Waterloo, 200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada (e-mail: juan.moreno-cruz@uwaterloo.ca). Wagner: Harvard University Center for the Environment, 26 Oxford Street, Cambridge, MA 02138 (e-mail: gwagner@fas.harvard.edu). Keith: Harvard John A. Paulson School Of Engineering and Applied Sciences and Harvard Kennedy School, 12 Oxford Street, Cambridge, MA 02138 (e-mail: david_keith@harvard.edu). Without any implications, we thank Cynthia Lin, Ken Gillingham, Kate Ricke, Soheil Shayegh, Karl Steininger, and Martin Weitzman for helpful comments and discussions. We thank the Weatherhead Initiative on Climate Engineering for support. All remaining errors are our own.

Conventional economic wisdom says that the optimal climate policy is to follow the logic of Pigou (1920) and price carbon dioxide (CO_2) and other greenhouse-gas emissions[1] at their marginal costs to society: internalize the negative externality, and get out of the way.[2] While Pigou is right, the conventional wisdom is wrong, or at least it is limiting. For one, it is limiting because of the unpriced, positive learning-by-doing externality inherent in the adoption of new, cleaner technologies (e.g., Acemoglu et al., 2012).[3] A second fundamental reason for why the conventional wisdom is wrong is that there is a long time delay between CO_2 emissions and their effects on welfare. The effects instead propagate through a long causal chain, with emissions affecting concentrations, concentrations affecting temperatures, and temperatures affecting damages affecting human welfare. Each link engenders its own possible intervention.

Society can avoid emitting CO_2 in the first place: *mitigation*. It can adjust to new climate realities: *adaptation*. It can extract carbon from the air: *carbon geoengineering*.[4] Lastly, it can attempt to affect climate outcomes directly: *solar geoengineering*.[5] The bulk of the climate economics literature focuses on mitigation (e.g., Acemoglu et al., 2012; Goulder and Pizer, 2008; Nordhaus, 2013; Stern, 2007), with some entries on adaptation (e.g., Bruin, Dellink and Tol, 2009; Kahn, 2013; Mendelsohn, 2012). Carbon geoengineering occupies a niche at once mundane and unique: economic models often fail to call it out because it merely looks like 'expensive mitigation'. It is not. In fact, it is the only intervention that allows for actually decreasing the stock of atmospheric CO_2 without simply waiting for slow, natural

[1]While there are important differences between long-lived climate forcers, like CO_2, and short-lived climate forcers like methane (Shindell et al., 2017), we here focus on CO_2, and henceforth use "CO_2" as a shortcut for greenhouse-gas emissions. Any mention of, e.g., "carbon stock" for expositional expediency should, thus, be interpreted as "CO_2 stock."

[2]Some invoke Coase (1960) instead of Pigou (1920), though Coase himself would likely agree that internalizing the negative carbon externalities all but requires a Pigouvian tax rather than a Coasian bargaining solution (Glaeser, Johnson and Shleifer, 2001).

[3]The existence of a second, positive externality and the policy interplay with CO_2 pricing leads to important political economy considerations (e.g., Acemoglu et al., 2016; Bennear and Stavins, 2007; Wagner et al., 2015; Meckling, Sterner and Wagner, 2017).

[4]Carbon geoengineering is commonly also referred to as 'carbon dioxide removal' (CDR). See NRC (2015*b*) for a survey of methods and their implications.

[5]Solar geoengineering, in turn, comes under various names including 'solar radiation management', 'albedo modification', 'climate remediation', and sometimes simply 'geoengineering' or 'climate engineering' as a catch-all term (e.g., Keith, 2000; NRC, 2015*a*).

processes to do so. The small economic literature on solar geoengineering, in turn, often focuses on it in isolation, with a few exceptions considering both solar geoengineering and mitigation as part of a mixed portfolio (e.g., Moreno-Cruz, 2015; Moreno-Cruz and Keith, 2012; Heutel, Moreno-Cruz and Shayegh, 2016). Our model attempts to capture the pertinent characteristics of each of these possible policy interventions in their most stylized form.

Mitigation is slow and relatively costly.[6] This, combined with inertia and non-locality, makes climate change the poster child of the free-rider problem, as countries and individuals seek to postpone emissions reduction measures with the intention of inducing higher mitigation efforts by others (e.g., Pigou, 1920; Cline, 1992; Cramton et al., 2017). We assume that the only way to create appropriate incentives for mitigation is via a broad-based CO_2 tax. In practice, that "tax" can take many forms and it alone is often far from optimal.[7] Mitigation alone, however, is not enough for an optimal solution, largely due to inertia in the climate system. Global average temperatures have already risen by around 1^{o}C since before the industrial revolution, with almost as much additional warming baked in due to elevated atmospheric CO_2 concentrations (IPCC, 2013; Friedlingstein et al., 2006). That points to the all-important time element in climate policy. It also highlights the importance of interventions further along the chain.

Carbon geoengineering mimics mitigation in important ways. In fact, for as long as emissions are not set to zero, there is no clear distinction on the effects of mitigation or carbon geoengineering in the climate-carbon system (Heutel, Moreno-Cruz and Ricke, 2016; NRC, 2015*b*). It is as slow as and costlier than mitigation—"slow," like mitigation, in the sense of propagating through the climate-economy chain, not necessarily in terms of the invest-

[6] "Costly," of course, is indeed relative. The question relevant for policy is costly compared to what? Substantial amounts of mitigation are cheap relatively to unmitigated climate change (Tol, 2018). Even with no net-negative emissions reductions opportunities, the costs of doing substantial mitigation are typically estimated to be no more than roughly 1 or 2% of global economic output (Nordhaus, 2017*a*,*b*). To put this number in perspective, the social costs of federal air and water regulations in the 1980s are estimated to be a few billion dollars, also in the order of 1% of US GDP at the time (Hazilla and Kopp, 1990).

[7] It could either take the form of a quantity-based instrument (Dales, 1968; Weitzman, 1974; Keohane, 2009), an implicit price instituted via other policy instruments (e.g., Bennear and Stavins, 2007), a direct tax (e.g., Metcalf, 2009), or a combination of two or more instruments (e.g., Pizer, 2002; Fankhauser, Hepburn and Park, 2010). More often than not, it comes in the form of deliberate technological interventions. See footnote 3.

ment decision.[8] Unlike mitigation alone, it can lead to net-negative changes in the atmospheric CO_2 stock in any given year, much faster than natural processes. Meanwhile, even if the world were to deploy mitigation and carbon geoengineering at scales leading to net negative emissions by mid-century, temperatures and sea levels would rise for decades and centuries to come (Matthews et al., 2009; Solomon et al., 2009), pointing to the need for potential further interventions down the climate system chain.

Solar geoengineering is quick, cheap, and imperfect (Keith, Parson and Morgan, 2010). It is quicker and, especially when looking at direct costs alone, cheaper than either mitigation or carbon geoengineering (NRC, 2015*a*). Solar geoengineering's direct 'engineering' costs are estimated to be roughly in the order of 0.01% of global economic output (McClellan, Keith and Apt, 2012; Moriyama et al., 2017), compared to roughly 1-2% associated with climate damages and with emissions mitigation.[9]

Solar geoengineering also intervenes further down the climate system chain. While that makes its impacts quicker, side-stepping the inertia inherent in the carbon cycle, it does not tackle excess CO_2 in the first place. Solar geoengineering is not anti-CO_2. It acts to directly offset excess radiative forcing, thus reducing climate changes. But even if solar geoengineering were to eliminate any net change in radiative forcing, climate changes will remain, as solar geoengineering's ability to compensate for all climate changes is inherently imperfect. Moreover, solar geoengineering entails additional environmental risks. We represent climate change through temperature T alone, and capture both the imperfect compensation and the additional risks in solar geoengineering's damage function.

Finally, there are large governance challenges. Solar geoengineering can be implemented without full participation (Barrett, 2008, 2014). Instead of sharing classic free-rider properties with mitigation and carbon geoengineering, solar geoengineering exhibits "free-driver" properties implying that the actor with the strongest preference for deployment wins (Wagner and Weitzman, 2012, 2015; Weitzman, 2015).[10] While solar geoengineering can

[8]Research on the cost of carbon geoengineering is still quite limited. There are considerable uncertainties, but there seems to be a consensus that for substantial amounts of carbon geoengineering, the costs will be higher than for the equivalent amount of mitigation. Here we assume the cost of carbon geoengineering is always higher than the cost of mitigation.

[9]See footnote 6.

[10]These "free-driver" properties have far-reaching implications, from the validity of

be under-supplied if the country with the means to implement it chooses not to do so (Moreno-Cruz and Smulders, 2017), low direct costs create the distinct possibility that solar geoengineering is oversupplied in the future. Thus, while a CO_2 tax, or its equivalent, is necessary to motivate mitigation and carbon geoengineering, a "temperature tax" is not.[11]

Adaptation, meanwhile, is imperfect and private. In fact, it is doubly imperfect, as it has no direct effect on either CO_2 stocks or on temperatures. While it affects provisions of public goods—from migration to mitigation—adaptation itself is typically rival and excludable, making it a classic private good (Samuelson, 1954).[12] Depending on the scale of adaptation, it can be relatively quick and cheap—think a second air conditioner—or slow and expensive—think moving entire cities to higher land (Desmet and Rossi-Hansberg, 2015). In any case, adaptation should not be confused with "suffering." Adaptation is deliberate (Kahn, 2013). Suffering, a loss in welfare because of inadequate climate policy interventions, is not.

Put back into the language around the climate-economic chain from emissions to human welfare, only mitigation propagates throughout the entire chain. The other three interventions are aimed at breaking otherwise believed-to-be firm links: carbon geoengineering breaks the link between emissions and concentrations; solar geoengineering breaks the link between concentrations and radiative forcing, in turn affecting temperatures and causing other climate impacts; adaptation breaks the link between climate impacts and damages. What then is the best way to combine these four instruments to optimally manage climate change?

To address this question, we develop a parsimonious model of climate change economics that captures the main trade-offs associated with all four instruments. Economic output, of which emissions are an important compo-

---

benefit-cost analyses in evaluating the role of solar geoengineering in optimal climate policy (Moreno-Cruz, 2015) to strategic coalition formations among nations (Ricke, Moreno-Cruz and Caldeira, 2013).

[11]The combination, a CO_2 tax pegged to temperatures (McKitrick, 2011), is similarly misguided for the simple reason that inherent inertia in the climate system delays feedback by decades or centuries. Tying a CO_2 tax to global average temperatures would be akin to tying a fire code implementation to the temperature of a house fire. It's disconnected in time from the underlying problem.

[12]Our model with one representative agent does not, in fact, lend itself to a proper analysis of this private goods aspect of adaptation. Doing so necessitates extending the framework to more than one agent.

nent[13], propagates through the entire emissions-concentrations-temperatures chain to damages, which, in turn, lead to reductions in economic output. Mitigation reduces emissions. Mitigation and carbon geoengineering reduce concentrations. Mitigation, carbon geoengineering, and solar geoengineering reduce temperatures. Mitigation, carbon geoengineering, solar geoengineering, and adaptation reduce the resulting damages.

Climate-economy models typically reduce both the climate and economic systems to their essential components. Nordhaus (1992, 2013)'s Dynamic Integrated Climate-Economy (DICE) model famously includes fewer than twenty main equations in order to calculate the optimal global CO_2 price path.[14] We reduce the climate system to a single dynamic equation to describe the accumulation of emissions in the atmosphere, S, and to a direct relation between global average temperatures, T, based on S at any given point, minus the effects of solar geoengineering.[15]

We use the fact that global average temperatures T are, to a first approximation, directly proportional to cumulative greenhouse gases in the atmosphere, our stock variable S. We employ what has commonly become known as a "cumulative emissions" model in climate science—to a first approximation, the resulting global average temperature is linear in cumulative emissions (Matthews et al., 2009; Matthews, Solomon and Pierrehumbert, 2012). That conclusion relies, in part, on the fact that most of the temperature response that will happen within a century due to added CO_2 in the atmosphere, happens within a decade.[16] We can take advantage of this T-S relation and resulting 'quasi-equilibrium' behavior of climate policy over the

[13]Breaking the link between economic output and emissions is itself an important goal of climate policy aimed at mitigating emissions in the first place. A natural extension of our model is to include two goods—one "dirty," one "clean"—and to model the substitutability among them (e.g., Acemoglu et al., 2012, 2016).

[14]See Nordhaus and Sztorc (2013) for extensive model documentation. For extensive critiques and long lists of well-known limitations, see, among others: Burke et al. (2016); Convery and Wagner (2015); Daniel, Litterman and Wagner (2016); Fisher and Le (2014); Kopp et al. (2016); Morgan and Keith (2008); Pindyck (2013); Stern (2013); Wagner and Weitzman (2015); Weitzman (2009*b*); NAS (2017).

[15]See, e.g., Nordhaus (1991); Golosov et al. (2014) for economic models incorporating a direct link between T and cumulative emissions, without the addition of solar geoengineering.

[16]"Maximum warming occurs about one decade after a carbon dioxide emission" (Ricke and Caldeira, 2014). Around half of global average warming due to a rapid increase in atmospheric CO_2 happens within a decade, whereas around a quarter happens after a century (Caldeira and Myhrvold, 2013).

time frames that matter for policy.[17]

The transient and equilibrium behavior of the climate system matters to solving our model. It also matters to the fundamental understanding of optimal climate policy. Instead of an optimal control problem with one knob—S—which is assumed to have a direct link to eventual temperature and climate outcomes over the long run, we now have a second, much quicker knob: T. While T depends on cumulative net emissions in the atmosphere, it can also be directly regulated via solar geoengineering. S and T, thus, affect economic welfare in distinct ways. Time plays an important role; so do benefits, costs, and risks. Breaking the direct link between S and T also immediately increases the number of policy goals beyond one. That alone all but guarantees that the "conventional wisdom" around a CO_2 tax needs to be overturned. More than one potential policy target calls for more than one policy intervention.[18]

1 General Framework

Focusing on the utility derived from $E(t)$, the consumption of fossil fuels and, thus, the emissions of CO_2, we assume our representative agent's utility function is quasilinear, given by:

$$U(E(t)) + Q_0(t). \tag{1}$$

This assumption is limiting in one important way: it does not allow us to distinguish between 'dirty' and 'clean' production and, thus, makes reductions in emissions necessarily costly—an oft-stated assumption in economics, albeit one worthy of further exploration.[19] $Q_0(t)$ is the consumption of all

[17]See Held et al. (2010) and Cao et al. (2015) on "fast" versus "slow" responses in the climate system, Proistosescu and Huybers (2017) on fast and slow modes of equilibrium climate sensitivity itself—"fast" here on geological timescales— and Nordhaus (1991) and Lemoine and Rudik (2017) for explicit discussions of time and the effects of inertia in climate-economic models. See also Ricke and Caldeira (2014) and Caldeira and Myhrvold (2013) for detailed modeling results. Caldeira and Myhrvold (2012) explore the implications of using temperature as a metric to evaluate climate and energy policies.

[18]Mundell (1968, p. 201), in reference to Tinbergen (1952), likens economic policy systems to "'overdetermined' or 'underdetermined' mathematical systems," unless the number of policy goals matches the number of instruments.

[19]See footnote 13.

other goods in the economy, taken to be the numeraire. The utility derived from consumption of fossil fuels is given by

$$U(E(t)) = \alpha E(t) - \frac{1}{2}\beta E(t)^2, \tag{2}$$

with $\alpha > 0$ and $\beta > 0$.

We consider a partial equilibrium model where global aggregate income, $Y(t)$, is exogenous and equal to $Q_0(t)$ plus the costs of fossil fuel consumption, $pE(t)$, damages from climate change, D, and costs of climate intervention, C:

$$Y(t) = pE(t) + Q(t) + D(T(t), S(t), G(t), A(t)) + C(R(t), G(t), A(t)), \tag{3}$$

Climate damages are denoted by $D(T(t), S(t), G(t), A(t))$ and are strictly increasing and weakly convex in T, S, and G:

$$D(T, S, G, A) = \frac{1}{2}\kappa\left(T^2 - 2\chi_A A\right) + \frac{1}{2}\sigma S^2 + \frac{1}{2}\gamma G^2, \tag{4}$$

with $\kappa > 0$, $\sigma > 0$, and $\gamma > 0$. Climate damages associated with global average surface temperature, $T(t)$, can be reduced with expenditures on adaptation, $A(t)$, where $\chi_A \geq 0$ models both the availability and effectiveness of adaptation measures. Climate damages further depend on S directly. Solar geoengineering, G, meanwhile enters both via T (see equation 8 below) and directly, in form of the damages associated with G.

The costs of managing the climate are given by $C(R(t), G(t), A(t))$, where $R(t)$ is the removal of CO_2 from the atmosphere: carbon geoengineering. Costs are assumed to be strictly increasing and convex in each element:

$$C(R, G, A) = \frac{1}{2}\nu R^2 + \frac{1}{2}\eta G^2 + \frac{1}{2}\omega A^2 \tag{5}$$

with $\nu > 0$, $\eta > 0$, $\omega > 0$. This cost function also assumes separability; to a first approximation this is likely to be true, but general equilibrium effects can create second order interactions.

Mitigation, $M(t)$, takes the form of reductions in emissions, $E(t)$, relative to a maximum level of emissions, $\bar{E}$, that maximizes utility where climate damages are not considered in the economy (see section 1.2 for derivations). Thus,

$$M(t) \equiv \bar{E} - E(t). \tag{6}$$

The costs of mitigation are measured in terms of forgone utility, given that $E(t)$ enters U in equation (2) directly. That is $C(M) = U(\bar{E}) - U(E)$, for any $E < \bar{E}$.[20]

1.1 Climate System Dynamics

We capture the climate system and its four-link chain from emissions via concentrations and temperatures to damages in two steps. The first dynamic equation represents the evolution of cumulative emissions of CO_2 in the atmosphere, $S(t)$:

$$\dot{S}(t) \equiv \frac{dS(t)}{dt} = E(t) - \chi_R R(t), \quad S(0) = S_0 > 0. \tag{7}$$

The stock of CO_2, $S(t)$, accumulates in the atmosphere with past emissions that result from the burning of fossil fuels, $E(t)$. Carbon geoengineering, $R(t)$, decreases $S(t)$, breaking the otherwise direct link between emissions and concentrations, with $\chi_R \geq 0$ showing if carbon geoengineering is available ($\chi_R > 0$) and how effective it is in reducing $S(t)$.

The second equation links global average temperature over time, $T(t)$, to cumulative CO_2 emissions via a linearized climate feedback parameter, λ:

$$T(t) = \lambda S(t) - \chi_G G(t). \tag{8}$$

Solar geoengineering, $G(t)$, enters the temperature equation linearly, with $\chi_G \geq 0$ showing its availability and effectiveness in reducing $T(t)$.

Equations (7) and (8) alone point to many possible extensions of our model, from more complex carbon-cycle dynamics introduced in some climate-economic models (e.g., Golosov et al., 2014), to a full treatment of inertia and how it translates into a "cumulative emissions" framework (e.g., Nordhaus, 1991; Lemoine and Rudik, 2017), to an explicit treatment of uncertainty (e.g., Moreno-Cruz and Keith, 2012; Heutel, Moreno-Cruz and Shayegh, 2016). Note also that our model introduces the full effects of solar geoengineering via both $T(t)$ and $G(t)$. $T(t)$ captures solar geoengineering's direct temperature impacts, proxying for radiative forcing. That representation alone

[20] We assume the costs, in terms of utility, of reducing emissions are lower than the equivalent reduction in emissions using carbon geoengineering. As it turns out, this assumption does not affect the behavior of the system as we describe below, but we make it explicit as it could affect the timing of the optimal policy in a calibrated model.

would diminish G's potentially positive effects on other dimensions, such as its direct carbon impact (Keith, Wagner and Zabel, 2017), but it alone would make solar geoengineering look 'too good'. First, tackling radiative forcing does not tackle CO_2. Second, G might have other potential negative impacts (e.g., Moreno-Cruz and Smulders, 2017). All are potentially important extensions of our work. Here we focus on this simple climate system and their most salient interactions to derive stylized facts and their implications.

1.2 Myopic scenario

The 'myopic' scenario assumes that there are no damages from climate change. The solution is trivial. It is also instructive, for two reasons. First, it introduces the panel of figures, 1a through 1d, we will use in subsequent sections to show the effects of various climate policy portfolios, building up to the most flexible option that allows for all four: mitigation M, adaptation A, carbon geoengineering R, and solar geoengineering G. Figure 1a shows the phase diagram pitting E versus S over time. While trivial, the system dynamics for the myopic scenario will be an important part of analysis as we expand the climate policy portfolio. Emissions in this myopic scenario are given by

$$E(t) = \bar{E} \equiv \frac{\alpha - p}{\beta}, \tag{9}$$

shown in Figure 1b. Carbon in the atmosphere accumulates linearly at a rate equal to $\bar{E}$:

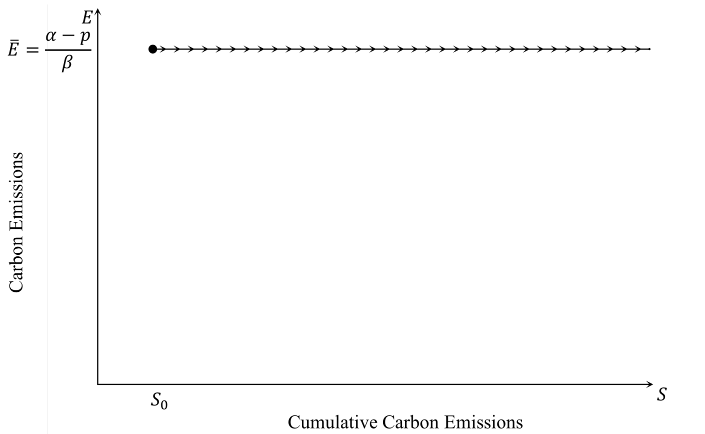
$$S(t) = S_0 + \bar{E}t, \tag{10}$$

shown in Figure 1c.

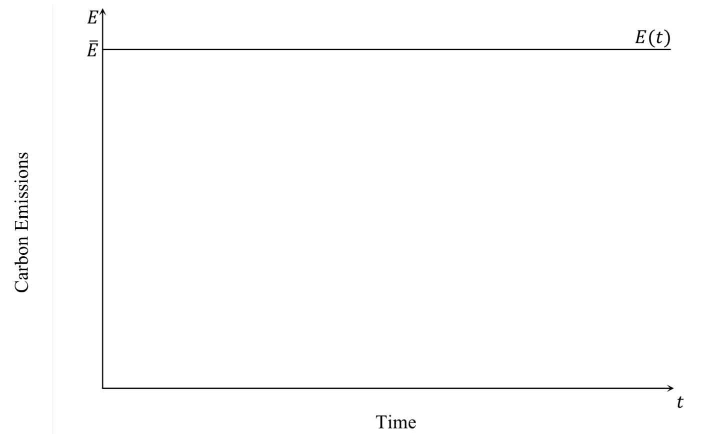
Second, this myopic solution is instructive because it shows where our model departs in important ways from prior economic analyses. Most prior climate-economic models assume the system approaches an equilibrium in the long term, even without mitigation (e.g., Nordhaus and Sztorc, 2013; Lemoine and Rudik, 2017). Our model instead is consistent with the most recent literature in the natural sciences, which suggests the resulting temperature is linear in cumulative emissions (Matthews et al., 2009; Matthews, Solomon and Pierrehumbert, 2012):

$$T(t) = \lambda S_0 + \lambda \bar{E}t, \tag{11}$$

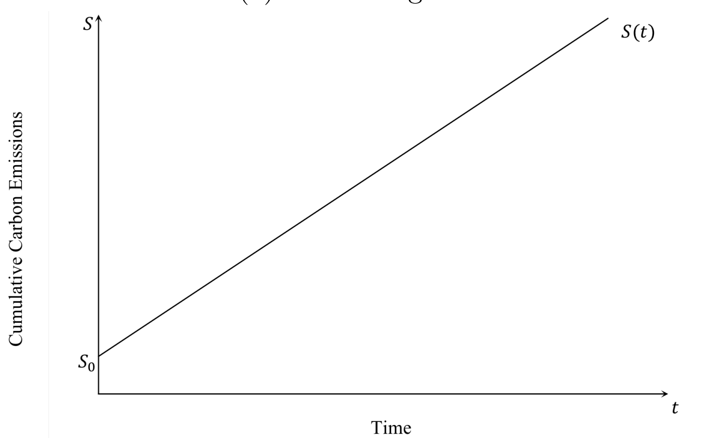
shown in Figure 1d.



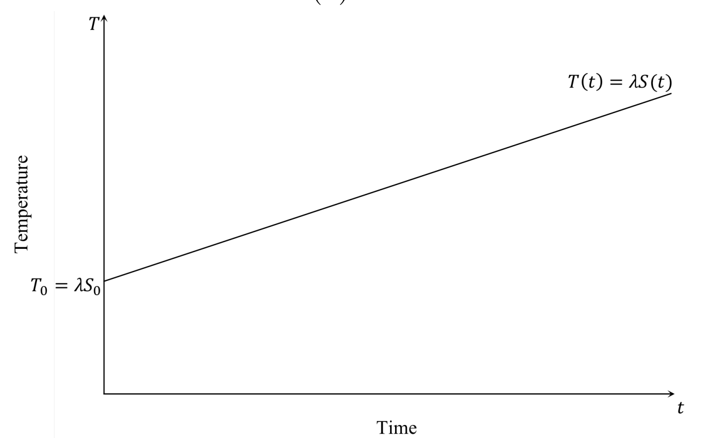
(a) Phase diagram



(b) E vs. t



(c) S vs. t



(d) T vs. t

Figure 1: Myopic scenario time trajectories and phase diagram. Panel (a) shows the phase diagram in an E vs. S plane. The line with arrows depicts the evolution of the system. Panels (b)-(d) show time trajectories for the variables of interest. Panel (b) shows emissions, E. Panel (c) shows cumulative emissions from some initial condition, S_0. Panel (d) shows temperature, T.

The myopic scenario shows one more important difference between a cumulative emissions model used here and prior concentrations equilibrium models. Here stopping emissions does not reduce temperatures—at least not at timescales relevant to policy. Stopping emissions, setting $E = 0$, merely stops temperatures from continuing to rise. While this conclusion is evident in the latest climate science literature focused on carbon budgets (e.g., Matthews et al., 2009; Matthews, Solomon and Pierrehumbert, 2012), it has not yet found its way into climate-economic models. One stark conclusion: The only way to reduce temperatures at timescales relevant to policy is by either removing carbon from the atmosphere, $R > 0$, or by reducing the amount of warming associated with any given amount of cumulative emissions, $G > 0$.

We do not want to dismiss the relevance of concentrations equilibrium models for showing eventual policy outcomes, nor the importance of pricing the risks associated with climatic extremes (e.g., Weitzman, 2009*b*). Their continued use in growth models is similarly appropriate for calculating the 'optimal' price of a ton of CO_2 emitted today (e.g., Nordhaus, 2013; Daniel, Litterman and Wagner, 2016). But it may be more appropriate and intuitive to move to a cumulative emissions model to highlight the roles of different climate policies and their respective trade-offs. In the subsequent sections, we analyze the system as we allow for more instruments to complement mitigation efforts.

2 Optimal Solution

The social planner maximizes the present discounted value of social welfare:

$$\max_{\{E,R,G,A\}} \int_0^\infty \{U(E(t)) + Q(t)\} e^{-\rho t} dt, \tag{12}$$

subject to the budget constraint (3), and equations (7) and (8). This four-equation system covers the full optimization problem. We further require that all instruments are non-negative; that is, $M \geq 0$, $R \geq 0$, $G \geq 0$, and $A \geq 0$.[21]

[21]Note that while E serves as our control variable, it is M defined by equation (6) that represents the mitigation instrument. With $E \geq 0$, $0 \leq M \leq \bar{E}$. Note also that from here on we drop time "(t)" for notational expediency and readability.

While T, per equation (8), unequivocally increases with S and decreases with G, S evolves according to a set of dynamic forces that require a look at the full optimization problem. The current value Hamiltonian is given by

$$\mathscr{H} = U(E) + Y - pE - D(T,S,G,A) - C(R,G,A) + \mu\left[E - \chi_R R\right],$$

where $\mu(t)$ is the co-state variable associated with the carbon emissions accumulation equation (7). We can now form a Lagrangian and extend the Hamiltonian to incorporate the non-negativity constraints:

$$\mathscr{L} = \mathscr{H} + \theta_E E + \theta_R R + \theta_G G + \theta_A A.$$

Using this formulation, the conditions for an optimal solution are given by:[22]

$$\frac{\partial \mathscr{L}}{\partial E} = U'(E) - p + \mu + \theta_E = 0, \tag{13}$$

$$\frac{\partial \mathscr{L}}{\partial R} = -C_R(R,G,A) - \chi_R \mu + \theta_R = 0, \tag{14}$$

$$\frac{\partial \mathscr{L}}{\partial G} = -D_T(T,S,G,A)T_G(S,G) - D_G(T,S,G,A) - C_G(R,G,A) + \theta_G = 0, \tag{15}$$

$$\frac{\partial \mathscr{L}}{\partial A} = D_A(T,S,G,A) - C_A(R,G,A) + \theta_A = 0, \tag{16}$$

$$\frac{\partial \mathscr{L}}{\partial S} = -D_T(T,S,G,A)T_S(S,G) - D_S(T,S,G,A) = \rho\mu - \dot{\mu}, \tag{17}$$

the complementary slackness conditions

$$\begin{aligned} E \geq 0, \theta_E \geq 0, E\theta_E = 0, \\ R \geq 0, \theta_R \geq 0, R\theta_R = 0, \\ G \geq 0, \theta_G \geq 0, G\theta_G = 0, \\ A \geq 0, \theta_A \geq 0, A\theta_A = 0, \end{aligned} \tag{18}$$

and the transversality condition,

$$\lim_{t\to\infty} e^{-\rho t}\mu S = 0. \tag{19}$$

It follows from equations (14)-(16) and the convexity assumptions for costs and damages, that θ_R, θ_G, and θ_A are always equal to zero; that is, R, G,

[22]We use the notation $F_x(x)$ to indicate $\partial F(x)/\partial x$.

and A are always strictly positive. This is not the case with emissions E; if the initial amount of cumulative carbon emissions is too high, carbon emissions could optimally be set to zero, $M = \bar{E}$ with negative emissions only possible if carbon geoengineering, R, is available. We analyze the optimal solution considering two regimes: positive ($E > 0$) and zero emissions ($E = 0$).

We can define the optimal CO_2 tax as:

$$\tau \equiv -\mu. \tag{20}$$

From (13) and (20) we find:

$$U'(E) + \theta_E = \alpha - \beta E + \theta_E = p + \tau. \tag{21}$$

This equation splits the world into two possible regimes, defined by τ relative to $\alpha - p$. We define two cases. With $\tau \leq \alpha - p$ the system enters the Positive-emissions regime. With $\tau > \alpha - p$ we move to the Zero-emissions regime. We discuss the two regimes in turn.

2.1 Positive-emissions regime

2.1.1 Mitigation

Following Kamien and Schwartz (1981) and especially Weitzman (2009*a*), we can already say a lot about the optimal solution. Equation (13) can now be written as

$$U'(E) = \rho + \tau, \tag{22}$$

reproducing the standard result that the marginal utility derived from emitting CO_2 into the atmosphere should equal the marginal cost of extracting fossil fuels, p, plus the optimal CO_2 tax. Replacing the function form, we get:

$$E = \bar{E} - \frac{\tau}{\beta}, \tag{23}$$

showing directly how emissions fall as the carbon tax increases. Conversely, restating equation (23) in terms of mitigation, we have $M = \tau/\beta$ increasing with τ. But mitigation is not the only instrument targeting CO_2 in the atmosphere. Carbon geoengineering R does, too, and arguably more directly.

2.1.2 Carbon geoengineering

Equation (14) immediately leads to:

$$R = \phi_R(\bar{E} - E), \tag{24}$$

where $\phi_R = \frac{\chi_R \beta}{\nu}$. Equation (24) shows directly that R is proportional to mitigation $R = \phi_R M$ and that its use increases as its variable costs, ν, fall. This result also shows that, for as long as emissions are positive, there is no difference in carbon geoengineering R and mitigation M. Both interventions are complementary and the net effect on cumulative emissions is additive.

Equations (23) and (24) also expand the 'conventional wisdom' presented in the introduction that emissions ought to be priced at their marginal cost to society. In the optimal solution, the marginal cost of carbon geoengineering R, too, equals the optimal carbon tax:

$$R = \phi_R \tau / \beta. \tag{25}$$

Conversely, assuming carbon geoengineering is available without any further binding restrictions its optimal use is guaranteed by an optimal carbon tax alone.[23]

2.1.3 Solar geoengineering

The most general optimal solution for solar geoengineering, G, follows a similar pattern. Equation (15) immediately leads to the conclusion that the total marginal costs of G—marginal damages plus marginal costs of implementation—are equal to the marginal reduction in temperature-induced damages:

$$D_G(T, S, G, A) + C_G(R, G, A) = -D_T(T, S, G, A) T_G(S, G). \tag{26}$$

[23] Note that the optimal CO_2 price is distinct from the social cost of carbon, SCC, even though the two often get conflated, and not merely because it should be the "SC-CO_2." The SCC is the marginal price of a ton of CO_2 given today's path (U.S. Government Interagency Working Group on Social Cost of Carbon, 2015). The SCC, thus, only equals the optimal CO_2 price, if one were to assume that today's path is optimal, a heroic assumption, to say the least. The interaction of carbon and solar geoengineering on the marginal (non-optimal) SCC is itself a potentially important, policy-relevant extension of this work. See, e.g., Kotchen (2016) for a framework that lends itself to this exploration.

Replacing assumed functional forms yields a direct relationship between solar geoengineering and cumulative carbon emissions:

$$G(S) = \phi_G \lambda S, \tag{27}$$

where $\phi_G = \frac{\chi_G \kappa}{\chi_G^2 \kappa + (\gamma + \eta)}$. The optimal CO_2 tax, thus, affects solar geoengineering through its effects on cumulative emissions that affect temperature further down the climate system chain, but the CO_2 tax does not set optimal solar geoengineering levels directly. Those instead depend on (26), a balance between direct costs and potential damages on the one hand with potential benefits on the other.

2.1.4 Adaptation

The final potential climate policy intervention is adaptation. Based on equation (16), here it is simply a constant balancing the marginal reduction in damages linked to temperature with the marginal costs of adaptation:

$$C_A(R, G, A) = -D_A(T, S, G, A),$$

which, after replacing our functional forms, implies:

$$A = \phi_A, \tag{28}$$

where $\phi_A = \frac{\chi_A \kappa}{\omega}$. Adaptation, thus, will not play an important role in the dynamics of the system. While this is the direct result of our assumptions, we do so intentionally to focus on the two more novel types of geoengineering interventions. Hence, in what follows, and to simplify our further discussion, we will refer to 'climate damages net of adaptation' simply as climate damages.

2.1.5 System dynamics

Replacing the functional forms in equation (17) and using (20) defines the behavior of the optimal carbon tax:

$$\rho \tau = \dot{\tau} + \left((1 - \chi_G \phi_G) \kappa \lambda^2 + \sigma \right) S. \tag{29}$$

Along the optimal path the present value of the carbon tax must equal the marginal reduction in future damages created by one extra unit of CO_2 in the atmosphere, plus the gains from not having to incur the costs of reducing that unit in the future.

Taking time derivatives of equation (23) and replacing the result in equation (29), we find the set of dynamic equations that govern the optimal solution when emissions are positive:

$$\dot{E} = \frac{1}{\beta}\left((1-\chi_G\phi_G)\kappa\lambda^2 + \sigma\right) S - \rho(\bar{E} - E) \tag{30}$$

$$\dot{S} = E - \chi_R\phi_R(\bar{E} - E) \tag{31}$$

The steady state, given by $\dot{E} = 0$ and $\dot{S} = 0$, is:

$$E^* = \frac{\chi_R\phi_R}{1+\chi_R\phi_R}\bar{E} \tag{32}$$

$$S^* = \frac{\bar{S}}{(1+\chi_R\phi_R)}, \tag{33}$$

where

$$\bar{S} = \frac{\bar{E}}{\frac{1}{\beta\rho}\left((1-\chi_G\phi_G)\kappa\lambda^2 + \sigma\right)}. \tag{34}$$

We will analyze the full implications below. For now, suffice it to emphasize that this does indeed capture the positive-emissions case, which extends to the steady state. With $\chi_R > 0$, steady-state emissions are allowed to remain above zero. If $\chi_R = 0$, the steady state is $E^* = 0$.

2.2 Zero-emissions regime

In the zero-emissions case, when $\tau > \alpha - p$, the system of dynamic equations is better expressed as a function of τ and S. Equation (31) now simplifies to:

$$\dot{S} = -\chi_R\phi_R\frac{\tau}{\beta}, \tag{35}$$

and the steady state of the system is characterized by $\tau^* = 0$ and $S^* = 0$.

To better understand the behavior of the system, we now introduce each possible climate policy intervention one by one as we build towards a full solution.

3 Analysis

3.1 Mitigation only

Suppose that only mitigation is available today; that is, assume $\chi_R = 0$, $\chi_G = 0$, and $\chi_A = 0$. To a first approximation, this situation represents the current state of climate policy. If $S(0) = S_{01} < \bar{S}$, we are in the Positive-Emissions regime—arguably unlike the current state of the world where atmospheric CO_2 concentrations are well above pre-industrial levels, albeit still below the oft-bandied political target for global average temperatures not to exceed '2^oC' above pre-industrial levels. The dynamics of the system are given by:

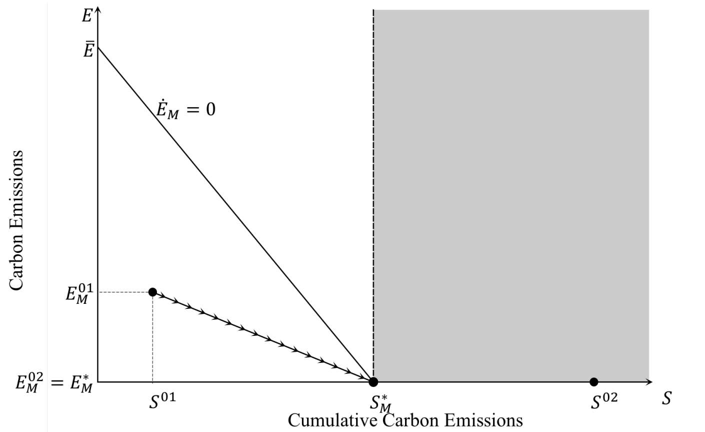
$$\dot{E} = \frac{1}{\beta}\left(\kappa\lambda^2 + \sigma\right) S - \rho(\bar{E} - E) \tag{36}$$

$$\dot{S} = E \tag{37}$$

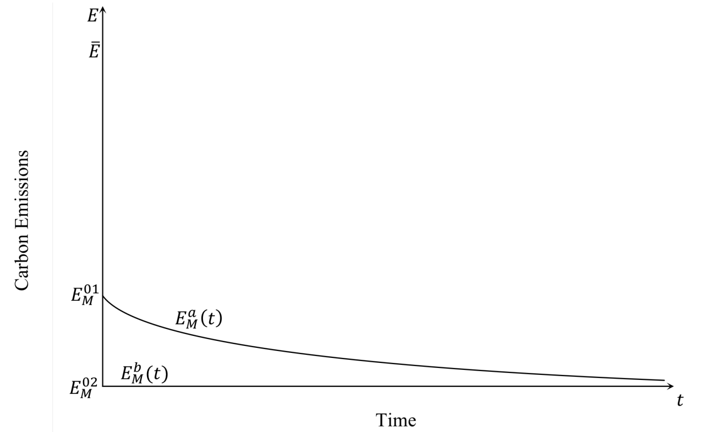
Note here, as throughout our analysis, the equivalency between $\bar{E} - E$ and M, as defined in equation (6). It is, in fact, mitigation M that is the climate policy instrument.

Figure 2a shows the evolution represented by equations (36) and (37). The system, unlike the myopic case, approaches a steady state in the long run where emissions are brought to zero, $E^*_M = 0$. The mitigation-only steady state has lower emissions and temperature compared to the business as usual scenario. Figure 2a also calls out the amount of mitigation, $M = \bar{E} - E$. Emissions fall, limiting cumulative emissions to the value $S^*_M = \bar{S}$. The carbon price would slowly increase towards $\bar{\tau} < \alpha - p$.

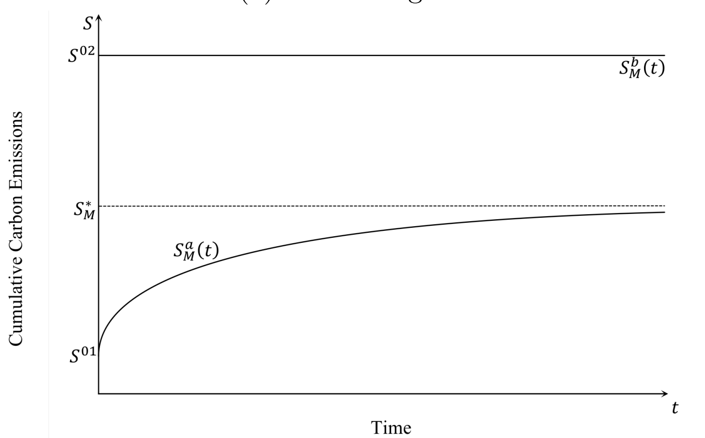
By comparison, $S(0) = S_{02} > \bar{S}$ instead defines the Zero-Emissions regime. Now emissions jump to zero and remain there for the rest of the planning horizon. Cumulative emissions stay at S_{02} and importantly, the carbon tax remains positive and equal to $\tau_{02} > \bar{\tau}$. Thus, while there is a positive willingness to reduce emissions, the non-negativity constraint on emissions means that the world is committed to an amount of cumulative emissions that is higher than what would be optimal if emissions were allowed to be negative. We next relax precisely this constraint, allowing for carbon geoengineering by setting $\chi_R > 0$.



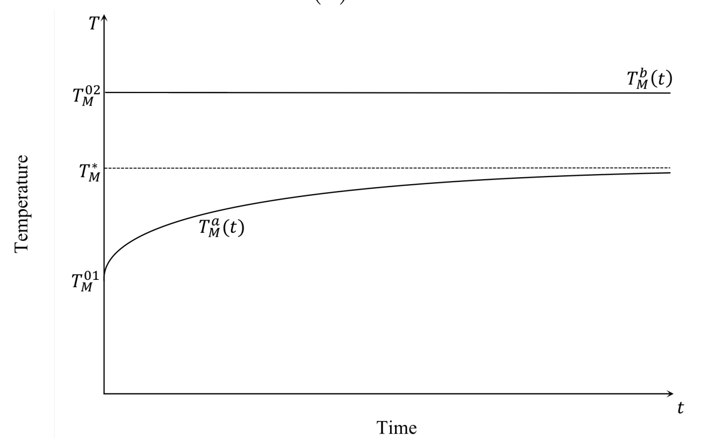
(a) Phase diagram



(b) E vs. t



(c) S vs. t



(d) T vs. t

Figure 2: Mitigation only time trajectories and phase diagram. The description and order of panel is as in Figure 1. The initial condition $S^{01} < S_M^*$ shows the behavior of the system in the Positive-emissions regime, while $S^{02} > S_M^*$ shows the Zero-emissions regime. The line with arrows shows that for any initial condition, emissions jump so the system approaches the steady state along the stable arm of the saddle. Mitigation is the difference between $\bar{E}$ and the emissions at time t. As shown in Panel (b), emissions approach zero as initial cumulative emissions are below S_M^*, but stay at zero otherwise. Similarly, Panels (c) and (d) show that cumulative emissions (temperature) either grow until they reach S_M^* (λS_M^*) or stay at S^{02} (λS^{02}).

3.2 Mitigation and Carbon Geoengineering

The introduction of carbon geoengineering, setting $\chi_R > 0$ while keeping $\chi_G = 0$ and $\chi_A = 0$, only affects the $\dot{S} = 0$ equation, which is now given by $\dot{S}_R = 0$:

$$\dot{S} = E - \chi_R \phi_R (\bar{E} - E), \tag{38}$$

with $\dot{E}$ still given by equation (36).

Mitigation and carbon geoengineering are substitutes with respect to their impact on atmospheric CO_2 stocks. That invokes popular discussion of potential 'moral hazard' or, more accurately 'crowding out', which is evident here with $R'(E) = -\phi_R < 0$. Introducing carbon geoengineering, thus, necessarily results in higher emissions and lower mitigation. The steady state is:

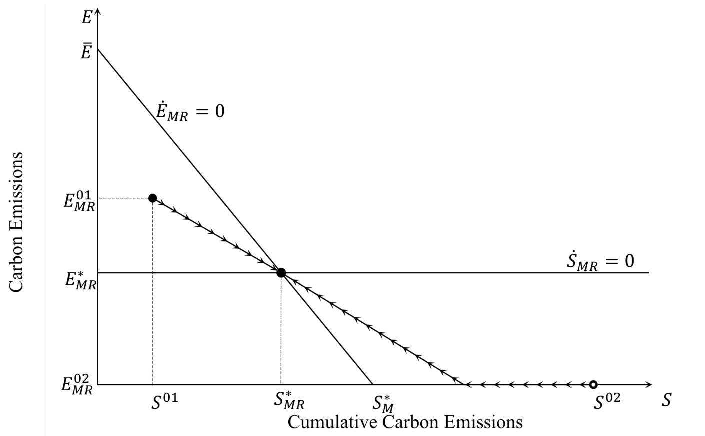
$$E^*_{MR} = \frac{\chi_R \phi_R}{1 + \chi_R \phi_R} \bar{E}, \tag{39}$$

which is clearly positive.

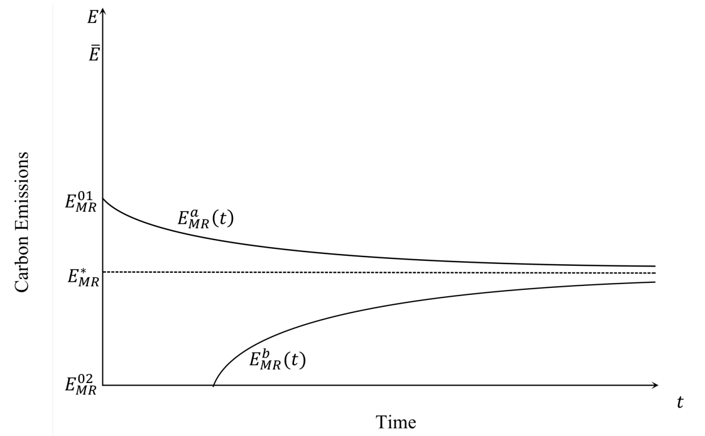
Figure 3, and in particular a direct comparison with the respective panels in Figure 2, shows the tradeoff between M and R: Carbon geoengineering allows for emissions to remain higher than without it being available (Panel 3b versus 2b). This alone would result in higher concentrations, were it not for carbon geoengineering in the first place. With carbon geoengineering, in fact, resulting atmospheric concentrations are lower (Panel 3c versus 2c).

Meanwhile, the climate variable closest to what ultimately feeds into the representative agent's utility function, temperatures T, is also lower with carbon geoengineering than without (Panel 3d versus 2d), allowing us to conclude qualitatively that welfare, as defined in this model, goes up with the availability of carbon geoengineering.

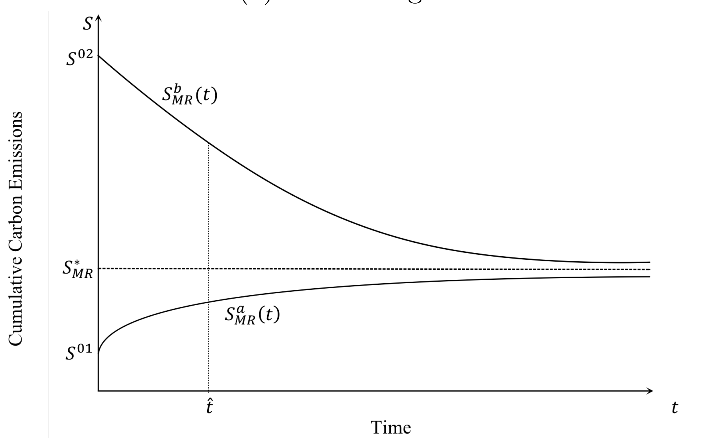
None of this includes potentially broader, behavioral tradeoffs, where, for example, the mere availability of carbon geoengineering might discourage moving to optimal mitigation levels in the first place—or the inverse, where the availability of one pushes more policy action on the other (e.g. Moreno-Cruz, 2015). We leave any such explorations for further models capable of incorporating behavioral aspects in a world of clearly sub-optimal climate policy.



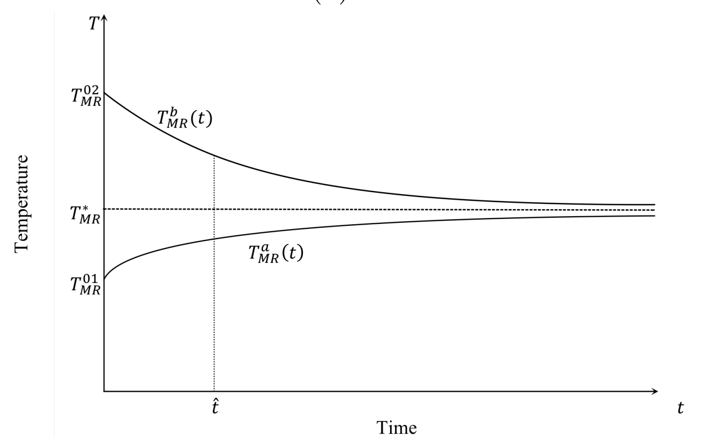
(a) Phase diagram



(b) E vs. t



(c) S vs. t



(d) T vs. t

Figure 3: Mitigation and carbon geoengineering time trajectories and phase diagram. The description and order of panel is as in Figure 1. The initial condition $S^{01} < S_{MR}^{*}$ shows the behavior of the system for Positive-emissions regime, while $S^{02} > S_{MR}^{*}$ shows the Zero-emissions regime. The line with arrows shows that for any initial condition, emissions jump so the system approaches the steady state along the stable arm of the saddle. As shown in Panel (b), emissions are always positive in steady state, but they either start positive and decline towards the steady state, or start at zero, and eventually increase towards the steady state, once R has reduced cumulative emissions enough. Similarly, Panels (c) and (d) show that cumulative emissions (temperature) either grow if their initial value is below S_{MR}^{*} (λS_{MR}^{*}) or decline towards this steady state otherwise.

3.3 Mitigation and Solar Geoengineering

Next consider the case where only mitigation and solar geoengineering are available, assuming $\chi_G > 0$ while $\chi_R = 0$ and $\chi_A = 0$. We do not claim for this to be a realistic climate policy scenario, where adaptation and carbon geoengineering surely ought to play a role, presumably before solar geoengineering enters the picture. It is still instructive to explore solar geoengineering in isolation with mitigation.

Adding solar geoengineering affects the dynamic equation (30) only:

$$\dot{E} = \frac{1}{\beta}\left((1 - \chi_G\phi_G)\kappa\lambda^2 + \sigma\right) S - \rho(\bar{E} - E), \tag{40}$$

while leaving the $\dot{S}$ equation unchanged. Any feedback from G to S, either via direct carbon feedback effects (Keith, Wagner and Zabel, 2017) or via policy or behavioral questions via mitigation efforts, is outside our model.

The steady state, when $\dot{E} = 0$ and $\dot{S} = 0$, is given by:

$$E^*_{MG} = 0 \tag{41}$$

$$S^*_{MG} = \frac{1}{\beta\rho}\left((1 - \chi_G\phi_G)\kappa\lambda^2 + \sigma\right) \equiv \bar{S} \tag{42}$$

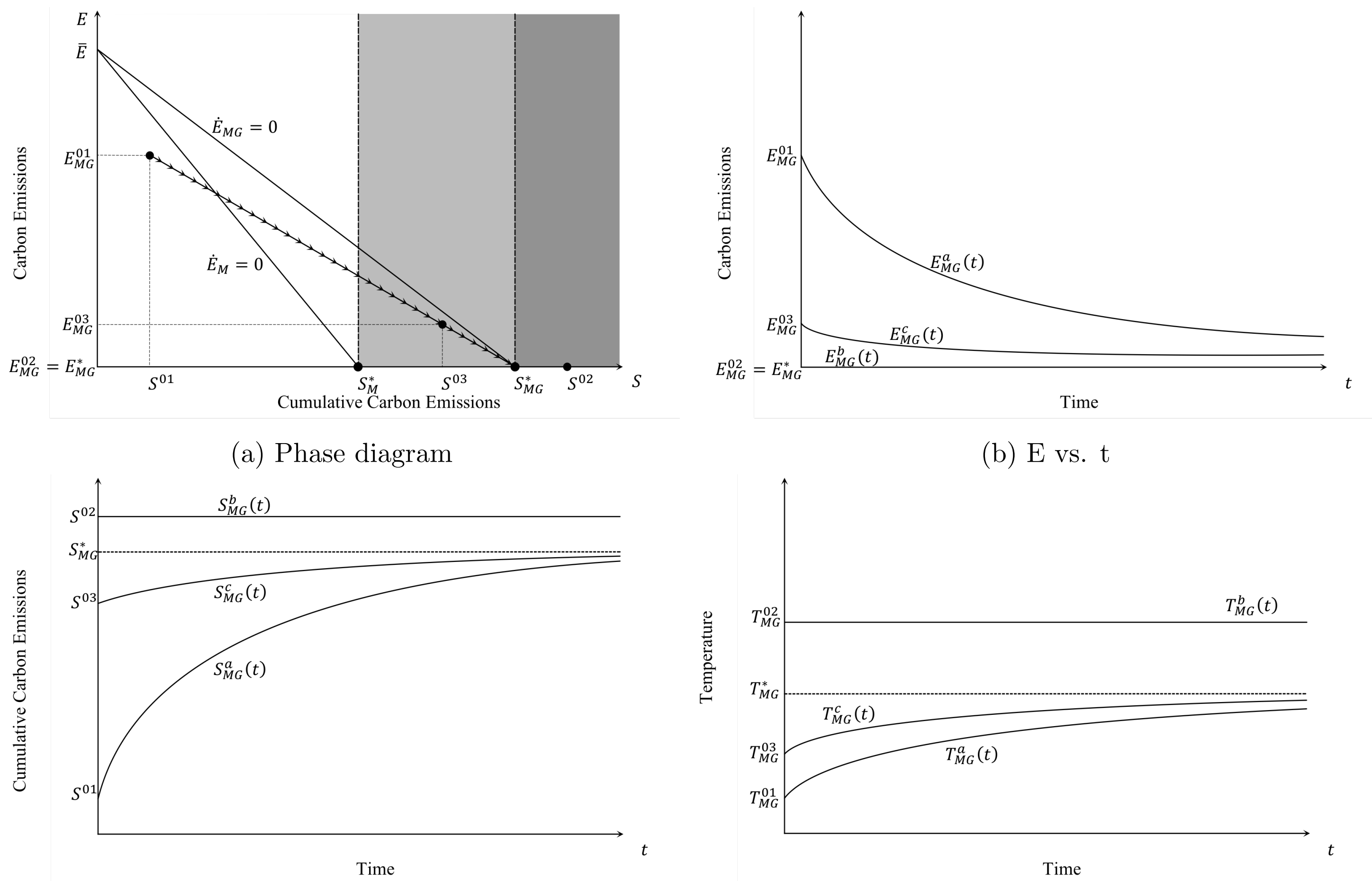
Introducing only solar geoengineering changes two elements in our system: First, the steady-state relation between emissions and concentrations rotates, reflecting a reduction in the marginal damage of each unit of emissions (Panel 4a).

The second important change is in the relation between temperature and carbon concentrations that changes from $T(S)$ to $T(S, G(S))$, making this relation less sensitive to cumulative carbon emissions. As soon as solar geoengineering is introduced, temperatures jump instantaneously to a lower level. This characteristic is what makes solar geoengineering unique among climate policy interventions: it creates a jump in what would otherwise be a state variable, breaking the firm link between S and T.

3.4 All Four Instruments

We are now ready to look at the interplay of all four instruments, when $\chi_R > 0$, $\chi_G > 0$, and $\chi_A > 0$.[24] Figure 5 summarizes the dynamics of the

[24] We did not discuss the implications of $\chi_A > 0$ in isolation. They are trivial given our modeling assumptions.



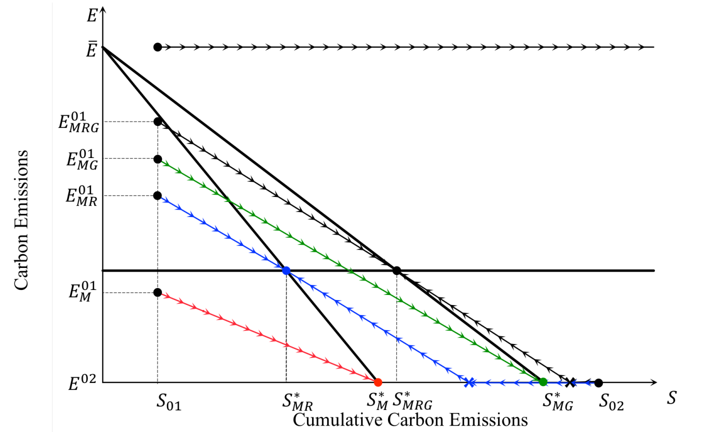
(a) Phase diagram

(b) E vs. t

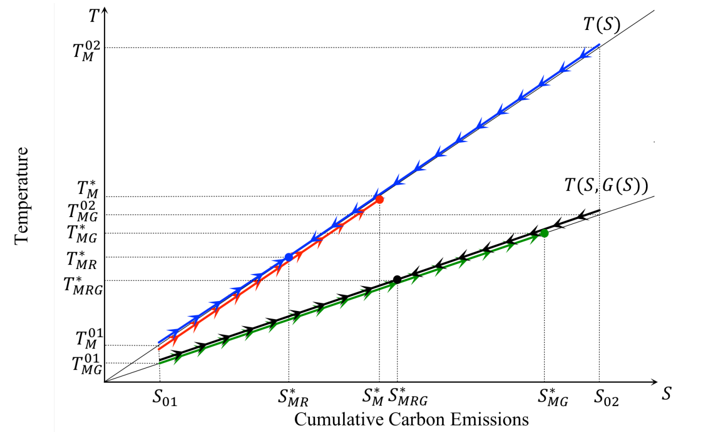
(c) S vs. t

(d) T vs. t

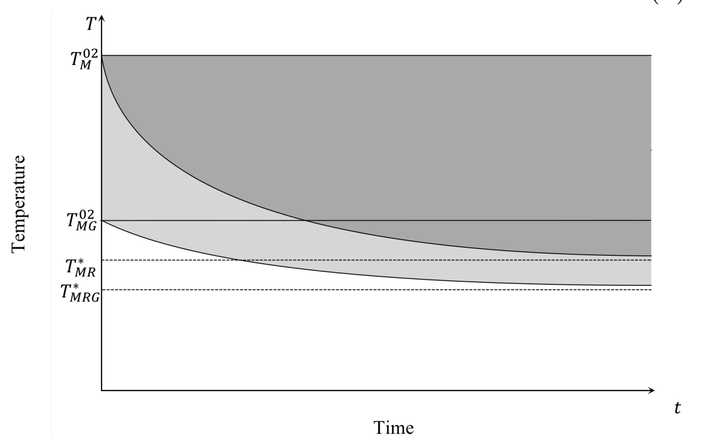
Figure 4: Mitigation and solar geoengineering time trajectories and phase diagram. The description and order of panels is as in Figure 1. This figure depicts three initial conditions. The initial condition $S^{01} < S^*_{MG}$ and $S^{03} < S^*_{MG}$ shows the behavior of the system for Positive-emissions regime, while $S^{02} > S^*_{MG}$ shows the Zero-emissions regime. The S^{03} is an example of the initial conditions that without G would lead to zero emissions, but with G keep emissions positive. The line with arrows shows that for any initial condition, emissions jump so the system approaches the steady state along the stable arm of the saddle. The behavior of the other panels is as in Figure 1 except that temperatures are reduced from λS to $\psi_G \lambda S$.



(a) E vs. S



(b) T vs. S



(c) T vs. t

Figure 5: All four climate policy instruments. Panel (a) shows the phase diagram in the E vs S space. Panel (b) show the phase diagram in the T vs S space.The black arrows show the transition towards the steady state when all instruments are used optimally. For comparison, the red arrows show the case of only mitigation, the blue arrows the case of mitigation and carbon geoengineering, and the green arrows the case of mitigation and solar geoengineering. Panel (c) shows the evolution of temperature over time for high cumulative emissions. It shows how solar geoengineering is initially used more than other instruments but decreases once carbon geoengineering increases. Not shown here are emissions over time, which are positive in steady state in this scenario.

system.

First, recall that, in our model, the introduction of solar geoengineering unambiguously results in higher emissions and, thus, higher concentrations, yet also lower temperatures. Meanwhile, the introduction of carbon geoengineering results in higher emissions, lower concentrations, and lower temperatures. When introducing both, the optimal level of cumulative emissions depends on the relative change in emissions. What is unambiguous is that temperatures decline (Panel 5b) and, thus, welfare, as defined in our model, increases. Note that while this conclusion includes negative externalities of unmitigated climate change on the one hand and of both carbon and solar geoengineering on the other, it also relies on the rational representative agent setup of our model. Within this framework, expanding the set of available climate policy interventions increases societal welfare.

One could argue that climate policies beyond mitigation are even more relevant in a situation where the planet has already overshot both concentrations and temperatures beyond their (long-term) equilibrium, needing to bring them down.[25] We show this scenario in Panel 4a. The effects are clear: emissions increase, while carbon geoengineering ensures that concentrations are falling immediately. The atmospheric CO_2 stock falls smoothly and slowly, as it approaches the new steady state. Something similar goes for having overshot temperature, with solar geoengineering as the instrument suitable for a possible direct intervention. Panel 5c shows the implications. First, all instruments need to be active whenever possible. Arbitrarily abstaining from any of them reduces welfare. Second, while solar geoengineering is used throughout, its share (depicted by the light-grey shaded area) declines over time, as the effects of mitigation and carbon geoengineering result in a declining temperature path. This suggest a role for solar geoengineering of 'buying time', implemented temporarily to maintain low temperatures as the world implements mitigation and carbon geoengineering. Once cumulative emissions have been brought down, solar geoengineering will continue to be used to the point where marginal costs of all instruments are equalized.

[25] Discussion of so-called "overshoot" scenarios has a long tradition in climate policy, going back at least to Broecker (2007).

4 Conclusions

This paper is at once easy and extremely difficult to summarize. It is easy to summarize because the main results are intuitive and supported by the canonical climate-economy model introduced here. It is difficult to summarize precisely because we attempt to introduce a basic taxonomy and canonical model that lends itself to exploring the most fundamental aspects of optimal climate policy.

The main contribution is reducing an incredibly complex problem to a canonical optimization problem represented by one dynamic equation that captures the (slow) evolution of atmospheric CO_2 and another that establishes the (fast) link between temperatures and cumulative emissions. While the scientific literature has increasingly looked to such cumulative emissions models to capture the most pertinent, short-term features of the emissions-temperature chain, most economic models to date rely on long-term equilibrium analyses. While important for other domains and questions, analysis focused on equilibria centuries hence strikes us as less important for climate policy, which is in its essence the challenge of managing the transition to a low-carbon, high-efficiency economy, not the challenge of managing a long-term equilibrium. While our focus on the linearized, short-term cumulative emissions-temperature relationship is surely a simplification of an otherwise complex climatic reality, we argue that it is a sensible way to break down the problem without missing the main characteristics of the all-important emissions-concentrations-temperatures-damages chain on the one hand, and of the basic anatomy of possible climate policy interventions on the other.

The contribution of this paper is to provide a framework that captures the essence of the climate-economy chain, allowing for a deeper exploration of the full set of climate policies: mitigation, carbon and solar geoengineering, and adaptation. All four interact in novel ways. For example, while conventional wisdom sets the CO_2 tax equal to marginal mitigation costs, this only applies when emissions are positive. Introducing carbon geoengineering shows that the tax ought to equal the marginal cost of carbon geoengineering, irrespective of whether emissions are positive or negative. Introducing either carbon or solar geoengineering leads to higher emissions but also to lower radiative forcing, the impacts of which are represented here by temperatures. This is an important tradeoff for climate policy conversations, where discussions often center around emissions reductions themselves, rather than eventual impacts.

We learn from our results and analysis, that mitigation alone does not lead to an optimal outcome. Moreover, in a world that is already at a level of suboptimally high cumulative CO_2 emissions, only carbon geoengineering can decrease atmospheric CO_2 concentrations without relying on natural processes to do so over the course of many decades and centuries, and during the transition to a world of lower CO_2 concentrations, only solar geoengineering can tackle excessive radiative forcing directly. Both possible interventions come with their own costs, potential damages, and large set of governance and political challenges.

This model allows—calls—for many an extension. One is expanding the model to include more than one representative agent. Another is introducing a "clean" good, in addition to the currently "dirty" one. A third is an explicit treatment of uncertainty. Ultimately, though, the real test for this model is how useful a guide it is for actual climate policy—as a substitute for or at least supplement to equilibrium-based climate-economy models.

References

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous. 2012. "The Environment and Directed Technical Change." *The American Economic Review*, 102(1): 131–166.

Acemoglu, Daron, Ufuk Akcigit, Douglas Hanley, and William Kerr. 2016. "Transition to Clean Technology." *Journal of Political Economy*, 124(1).

Barrett, Scott. 2008. "The Incredible Economics of Geoengineering." *Environmental and Resource Economics*, 39(1): 45–54.

Barrett, Scott. 2014. "Solar Geoengineerings Brave New World: Thoughts on the Governance of an Unprecedented Technology." *Review of Environmental Economics and Policy*, 8(2): 249–269.

Bennear, Lori Snyder, and Robert N. Stavins. 2007. "Second-best theory and the use of multiple policy instruments." *Environmental and Resource Economics*, 37(1): 111–129.

Broecker, Wallace S. 2007. "CO_2 Arithmetic." *Science*, 315(5817): 1371–1371.

Bruin, Kelly C. de, Rob B. Dellink, and Richard S. J. Tol. 2009. "AD-DICE: an implementation of adaptation in the DICE model." *Climatic Change*, 95(1-2): 63–81.

Burke, M., M. Craxton, C. D. Kolstad, C. Onda, H. Allcott, E. Baker, L. Barrage, R. Carson, K. Gillingham, J. Graff-Zivin, M. Greenstone, S. Hallegatte, W. M. Hanemann, G. Heal, S. Hsiang, B. Jones, D. L. Kelly, R. Kopp, M. Kotchen, R. Mendelsohn, K. Meng, G. Metcalf, J. Moreno-Cruz, R. Pindyck, S. Rose, I. Rudik, J. Stock, and R. S. J. Tol. 2016. "Opportunities for advances in climate change economics." *Science*, 352(6283): 292–293.

Caldeira, K., and N. P. Myhrvold. 2013. "Projections of the pace of warming following an abrupt increase in atmospheric carbon dioxide concentration." *Environmental Research Letters*, 8(3): 034039.

Caldeira, Ken, and Nathan P. Myhrvold. 2012. "Temperature change vs. cumulative radiative forcing as metrics for evaluating climate consequences of energy system choices." *Proceedings of the National Academy of Sciences*, 109(27): E1813–E1813.

Cao, Long, Govindasamy Bala, Meidi Zheng, and Ken Caldeira. 2015. "Fast and slow climate responses to CO2 and solar forcing: A linear multivariate regression model characterizing transient climate change." *Journal of Geophysical Research: Atmospheres.*

Cline, W. R. 1992. *The economics of global warming.* Institute for International Economics.

Coase, Ronald H. 1960. "The Problem of Social Cost." In *Classic Papers in Natural Resource Economics.* , ed. Chennat Gopalakrishnan, 87–137. Palgrave Macmillan UK.

Convery, Frank J., and Gernot Wagner. 2015. "ReflectionsManaging Uncertain Climates: Some Guidance for Policy Makers and Researchers." *Review of Environmental Economics and Policy*, 9(2): 304–320.

Cramton, Peter, David J.C. MacKay, Axel Ockenfels, and Steven Stoft. 2017. *Global Carbon Pricing: The Path to Climate Cooperation.* MIT Press.

Dales, John Harkness. 1968. *Pollution, Property & Prices: An Essay in Policy-making and Economics.* University of Toronto Press. Google-Books-ID: No2GkSzFCikC.

Daniel, Kent, Robert B. Litterman, and Gernot Wagner. 2016. "Applying asset pricing theory to calibrate the price of climate risk." Working Paper.

Desmet, Klaus, and Esteban Rossi-Hansberg. 2015. "On the spatial economic impact of global warming." *Journal of Urban Economics*, 88: 16–37.

Fankhauser, Samuel, Cameron Hepburn, and Jisung Park. 2010. "Combining multiple climate policy instruments: how not to do it." *Climate Change Economics*, 01(03): 209–225.

Fisher, Anthony C., and Phu V. Le. 2014. "Climate Policy: Science, Economics, and Extremes." *Review of Environmental Economics and Policy*, 8(2): 307–327.

Friedlingstein, P., P. Cox, R. Betts, L. Bopp, W. von Bloh, V. Brovkin, P. Cadule, S. Doney, M. Eby, I. Fung, G. Bala, J. John, C. Jones, F. Joos, T. Kato, M. Kawamiya, W. Knorr, K. Lindsay, H. D. Matthews, T. Raddatz, P. Rayner, C. Reick, E. Roeckner, K.-G. Schnitzler, R. Schnur, K. Strassmann, A. J. Weaver, C. Yoshikawa, and N. Zeng. 2006. "ClimateCarbon Cycle Feedback Analysis: Results from the C4MIP Model Intercomparison." *Journal of Climate*, 19(14): 3337–3353.

Glaeser, Edward, Simon Johnson, and Andrei Shleifer. 2001. "Coase Versus the Coasians." *The Quarterly Journal of Economics*, 116(3): 853–899.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski. 2014. "Optimal Taxes on Fossil Fuel in General Equilibrium." *Econometrica*, 82(1): 41–88.

Goulder, Lawrence H., and William A. Pizer. 2008. "The Economics of Climate Change." In *The New Palgrave Dictionary of Economics.* . 2nd edition ed. Palgrave MacMillan.

Hazilla, Michael, and Raymond J. Kopp. 1990. "Social Cost of Environmental Quality Regulations: A General Equilibrium Analysis." *Journal of Political Economy*, 98(4): 853–873.

Held, Isaac M., Michael Winton, Ken Takahashi, Thomas Delworth, Fanrong Zeng, and Geoffrey K. Vallis. 2010. "Probing the Fast and Slow Components of Global Warming by Returning Abruptly to Preindustrial Forcing." *Journal of Climate*, 23(9): 2418–2427.

Heutel, Garth, Juan Moreno-Cruz, and Katharine Ricke. 2016. "Climate Engineering Economics." *Annual Review of Resource Economics*, 8(1): 99–118.

Heutel, Garth, Juan Moreno-Cruz, and Soheil Shayegh. 2016. "Climate tipping points and solar geoengineering." *Journal of Economic Behavior & Organization*, 132: 19–45.

IPCC. 2013. "Fifth Assessment Report: Climate Change."

Kahn, Matthew E. 2013. *Climatopolis: How Our Cities Will Thrive in the Hotter Future.* Basic Books. Google-Books-ID: owQXBQAAQBAJ.

Kamien, Morton I., and Nancy Lou Schwartz. 1981. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management.* New York:North-Holland. Google-Books-ID: 0IoGUn8wjDQC.

Keith, David W. 2000. "Geoengineering the climate: history and prospect." *Annual Review of Energy and the Environment*, 25(1): 245–284.

Keith, David W., Edward Parson, and M. Granger Morgan. 2010. "Research on global sun block needed now." *Nature*, 463: 426–427.

Keith, David W., Gernot Wagner, and Claire L. Zabel. 2017. "Potentially large carbon impact from solar geoengineering."

Keohane, Nathaniel O. 2009. "Cap and Trade, Rehabilitated: Using Tradable Permits to Control U.S. Greenhouse Gases." *Review of Environmental Economics and Policy*, 3(1): 42–62.

Kopp, Robert E., Rachael Shwom, Gernot Wagner, and Jiacan Yuan. 2016. "Tipping elements and climate-economic shocks: Pathways toward integrated assessment." *Earth's Future*, 4(8): 346–372.

Kotchen, Matthew J. 2016. "Which Social Cost of Carbon? A Theoretical Perspective." National Bureau of Economic Research Working Paper 22246.

Lemoine, Derek, and Ivan Rudik. 2017. "Steering the Climate System: Using Inertia to Lower the Cost of Policy." *American Economic Review*, 107(10): 2947–2957.

Matthews, H. Damon, Nathan P. Gillett, Peter A. Stott, and Kirsten Zickfeld. 2009. "The proportionality of global warming to cumulative carbon emissions." *Nature*, 459(7248): 829–832.

Matthews, H. Damon, Susan Solomon, and Raymond Pierrehumbert. 2012. "Cumulative carbon as a policy framework for achieving climate stabilization." *Phil. Trans. R. Soc. A*, 370(1974): 4365–4379.

McClellan, Justin, David W. Keith, and Jay Apt. 2012. "Cost analysis of stratospheric albedo modification delivery systems." *Environmental Research Letters*, 7(3): 034019.

McKitrick, Ross. 2011. "A simple state-contingent pricing rule for complex intertemporal externalities." *Energy Economics*, 33(1): 111–120.

Meckling, Jonas, Thomas Sterner, and Gernot Wagner. 2017. "Policy sequencing toward decarbonization." *Nature Energy*, 1.

Mendelsohn, Robert. 2012. "The economics of adaptation to climate change in developing countries." *Climate Change Economics*, 03(02): 1250006.

Metcalf, Gilbert E. 2009. "Designing a Carbon Tax to Reduce U.S. Greenhouse Gas Emissions." *Review of Environmental Economics and Policy*, 3(1): 63–83.

Moreno-Cruz, Juan B. 2015. "Mitigation and the geoengineering threat." *Resource and Energy Economics*, 41: 248–263.

Moreno-Cruz, Juan B., and David W. Keith. 2012. "Climate policy under uncertainty: a case for solar geoengineering." *Climatic Change*, 121(3): 431–444.

Moreno-Cruz, Juan B., and Sjak Smulders. 2017. "Revisiting the economics of climate change: the role of geoengineering." *Research in Economics*, 71(2): 212–224.

Morgan, M. Granger, and David W. Keith. 2008. "Improving the way we think about projecting future energy use and emissions of carbon dioxide." *Climatic Change*, 90(3): 189–215.

Moriyama, Ryo, Masahiro Sugiyama, Atsushi Kurosawa, Kooiti Masuda, Kazuhiro Tsuzuki, and Yuki Ishimoto. 2017. "The cost of stratospheric climate engineering revisited." *Mitigation and Adaptation Strategies for Global Change*, 22(8): 1207–1228.

Mundell, Robert A. 1968. *International Economics.* Macmillan.

NAS. 2017. *Valuing Climate Damages: Updating Estimation of the Social Cost of Carbon Dioxide.*

Nordhaus, William D. 1991. "To Slow or Not to Slow: The Economics of The Greenhouse Effect." *The Economic Journal*, 101(407): 920–937.

Nordhaus, William D. 1992. "An Optimal Transition Path for Controlling Greenhouse Gases." *Science*, 258(5086): 1315–1319.

Nordhaus, William D. 2013. *The Climate Casino: Risk, Uncertainty, and Economics for a Warming World.* Yale University Press.

Nordhaus, William D. 2017*a*. "Evolution of Assessments of the Economics of Global Warming: Changes in the DICE model, 1992 2017." National Bureau of Economic Research Working Paper 23319.

Nordhaus, William D. 2017*b*. "Revisiting the social cost of carbon." *Proceedings of the National Academy of Sciences*, 114(7): 1518–1523.

Nordhaus, William D., and Paul Sztorc. 2013. *DICE 2013R: Introduction and Users Manual.*

NRC. 2015*a*. *Climate Intervention: Carbon Dioxide Removal and Reliable Sequestration.* Washington, D.C.:National Academies Press.

NRC. 2015*b*. "Climate Intervention: Reflecting Sunlight to Cool Earth." National Academies Press.

Pigou, A. C. 1920. *The Economics of Welfare.* Palgrave Macmillan.

Pindyck, Robert S. 2013. "Climate Change Policy: What Do the Models Tell Us?" *Journal of Economic Literature*, 51(3): 860–872.

Pizer, William A. 2002. "Combining price and quantity controls to mitigate global climate change." *Journal of Public Economics*, 85(3): 409–434.

Proistosescu, Cristian, and Peter J. Huybers. 2017. "Slow climate mode reconciles historical and model-based estimates of climate sensitivity." *Science Advances*, 3(7): e1602821.

Ricke, Katharine L., and Ken Caldeira. 2014. "Maximum warming occurs about one decade after a carbon dioxide emission." *Environmental Research Letters*, 9(12): 124002.

Ricke, Katharine L., Juan B. Moreno-Cruz, and Ken Caldeira. 2013. "Strategic incentives for climate geoengineering coalitions to exclude broad participation." *Environmental Research Letters*, 8(1): 014021.

Samuelson, Paul A. 1954. "The pure theory of public expenditure." *The review of economics and statistics*, 387–389.

Shindell, D., N. Borgford-Parnell, M. Brauer, A. Haines, J. C. I. Kuylenstierna, S. A. Leonard, V. Ramanathan, A. Ravishankara, M. Amann, and L. Srivastava. 2017. "A climate policy pathway for near- and long-term benefits." *Science*, 356(6337): 493–494.

Solomon, Susan, Gian-Kasper Plattner, Reto Knutti, and Pierre Friedlingstein. 2009. "Irreversible climate change due to carbon dioxide emissions." *Proceedings of the National Academy of Sciences*, 106(6): 1704–1709.

Stern, Nicholas. 2013. "The Structure of Economic Modeling of the Potential Impacts of Climate Change: Grafting Gross Underestimation of Risk onto Already Narrow Science Models." *Journal of Economic Literature*, 51(3): 838–859.

Stern, Nicholas Herbert. 2007. *The Economics of Climate Change: The Stern Review.* Cambridge University Press.

Tinbergen, Jan. 1952. *On the theory of economic policy.* North-Holland.

Tol, Richard SJ. 2018. "The Economic Impacts of Climate Change." *Review of Environmental Economics and Policy*, 12(1): 4–25.

U.S. Government Interagency Working Group on Social Cost of Carbon. 2015. "Technical Update of the Social Cost of Carbon for Regulatory Impact Analysis Under Executive Order 12866."

Wagner, Gernot, and Martin L. Weitzman. 2012. "Playing God." *Foreign Policy.*

Wagner, Gernot, and Martin L. Weitzman. 2015. *Climate Shock: The Economic Consequences of a Hotter Planet.* Princeton University Press.

Wagner, Gernot, Tomas Kberger, Susanna Olai, Michael Oppenheimer, Katherine Rittenhouse, and Thomas Sterner. 2015. "Energy policy: Push renewables to spur carbon pricing." *Nature*, 525(7567): 27–29.

Weitzman, Martin L. 1974. "Prices vs. quantities." *The review of economic studies*, 41(4): 477–491.

Weitzman, Martin L. 2009*a*. *Income, Wealth, and the Maximum Principle.* Harvard University Press.

Weitzman, Martin L. 2009*b*. "On Modeling and Interpreting the Economics of Catastrophic Climate Change." *Review of Economics and Statistics*, 91(1): 1–19.

Weitzman, Martin L. 2015. "A Voting Architecture for the Governance of Free-Driver Externalities, with Application to Geoengineering." *The Scandinavian Journal of Economics*, 117(4): 1049–1068.